Vectors Review #2

① a) $L_2: \; r = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$

b) If L_1 & L_3 are perpendicular, then $\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix} = 0$

$$2(-7) + 1(-2) - 8(k) = 0$$
$$-16 - 8k = 0$$
$$-8k = 16$$
$$k = -2$$

c) $-3 + 2p = 5 - 7q$

$-1 + p = 0 - 2q \rightarrow p = 1 - 2q$

$$-3 + 2(1 - 2q) = 5 - 7q$$
$$-3 + 2 - 4q = 5 - 7q$$
$$3q = 6$$
$$q = 2$$

$\begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -7 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} -9 \\ -4 \\ -1 \end{pmatrix}$ so $A = (-9, -4, -1)$

d) Remember that $B = (-8, -5, 25)$

Let $C = (x, y, z)$, so $\overrightarrow{BC} = \begin{pmatrix} x-(-8) \\ y-(-5) \\ z-25 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$ ← given

i) $\overrightarrow{AB} = \begin{pmatrix} -8-(-9) \\ -5-(-4) \\ 25-(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 26 \end{pmatrix}$

so $x = -2$, $y = -2$, $z = 1$

$C = (-2, -2, 1)$

ii) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\begin{pmatrix} 1 \\ -1 \\ 26 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix}$

$|\overrightarrow{AC}| = \sqrt{7^2 + 2^2 + 2^2}$
$= \sqrt{57}$

(2) a) $\overrightarrow{AB} = \begin{pmatrix} 1-3 \\ 5-2 \\ 3-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

b) $|\overrightarrow{AB}| = \sqrt{(-2)^2+3^2+2^2} = \sqrt{17}$

unit vector : $\frac{1}{\sqrt{17}} \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

c) $\frac{2}{\sqrt{17}} \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

(3) $\cos\theta = \frac{3(4)+4(-5)+5(-3)}{\sqrt{50} \cdot \sqrt{50}} = \frac{-23}{50}$

* Read the question carefully!

(4) a) $L_1 : r = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

b) If $s=2$, then $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$, so

A lies on L_2.

8 typo!

c) In other words, show that $B = (\cancel{9}, 1, 14)$

↑ intersection of L_1 & L_2

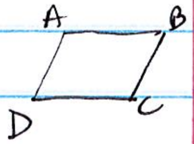
i) $8 + 0t = 2 + 2s$

$6 = 2s$

$3 = s$ → when $s = 3$ on L_2, it should be the intersection.

$\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix} = \overrightarrow{OB}$

ii) $\overrightarrow{AB} = \begin{pmatrix} 8-6 \\ 1-2 \\ 14-9 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$



d) Since ABCD is a parallelogram, $\overrightarrow{AB} = \overrightarrow{DC}$

so $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2-x \\ 1-y \\ -4-z \end{pmatrix}$ $\left.\begin{array}{l} x = 0 \\ y = 2 \\ z = -9 \end{array}\right\}$ so $D = (0, 2, -9)$

and

$\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ -9 \end{pmatrix}$

(5) $2 + s = 3 - t \rightarrow s = 1 - t$

$5 + 2s = -3 + 3t$

$5 + 2(1-t) = -3 + 3t$

$5 + 2 - 2t = -3 + 3t$

$7 - 2t = -3 + 3t$

$10 = 5t$

$2 = t$

have to plug into r_2

$\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ so $T = (1, 3, 0)$

(6) a) $\overrightarrow{AB} = \begin{pmatrix} 3-(-1) \\ 6-3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -4-(-1) \\ 4-3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 4(-3) + 3(1) = -9$

b) $\cos\theta = \dfrac{-9}{\sqrt{25} \cdot \sqrt{10}}$

$\theta = 125^\circ$ (3 sig figs)

(7) $P = (-5, 11, -8)$ $\quad Q = (-4, 9, -5)$

a) i) $\overrightarrow{PQ} = \begin{pmatrix} -4-(-5) \\ 9-11 \\ -5-8 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ or $i - 2j + 3k$

ii) $L_1 = \begin{pmatrix} -5 \\ 11 \\ -8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = (-5+s)i + (11-2s)j + (-8+3s)k$

b) $2 = -5+s$ $\quad 11-2(7) = y_1$ $\quad -8+3(7) = z_1$

$7 = s$ $\quad -3 = y_1$ $\quad 13 = z_1$

c) L_2 can also be written as $r = \begin{pmatrix} 2 \\ 9 \\ 13 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$-5 + 1s = 2+t \rightarrow s = 7+t$

$11 - 2s = 9+2t$

$11 - 2(7+t) = 9+2t$

$11 - 14 - 2t = 9+2t$

$-4t = 12$

$t = -3$

at $t=3$, $r = \begin{pmatrix} 2 \\ 9 \\ 13 \end{pmatrix} + -3\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$ or $-i + 3j + 4k$

d) $\cos\theta = \dfrac{1(1) - 2(2) + 3(3)}{\sqrt{14} \cdot \sqrt{14}}$

$\theta = 64.6^\circ$