Vectors Review #1

① $\overrightarrow{AB} = \begin{pmatrix} 4-2 \\ -2-(-3) \\ -5-(-1) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

② $|\overrightarrow{AB}|$ = magnitude $= \sqrt{2^2+1^2+(-4)^2} = \sqrt{21}$

③ Unit vector: $\dfrac{1}{\sqrt{21}}\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

④ $\sqrt{(2k)^2+(1k)^2+(-4k)^2} = 10$

$\sqrt{21k^2} = 10$

$21k^2 = 100$

$k^2 = \dfrac{100}{21}$

$k = \sqrt{\dfrac{100}{21}} = \dfrac{10}{\sqrt{21}}$

So vector is $\dfrac{10}{\sqrt{21}}\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

⑤ $B = (4,-2,-5)$. Use $(0,0,-5)$ on z-axis.

distance $= \sqrt{(4-0)^2+(-2-0)^2+(-5--5)^2} = \sqrt{20}$ or $2\sqrt{5}$

⑥ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

↳ I used point A, but you could also use B.

⑦ $u \cdot v = -2(-5) + 3(-2) + -3(4) = -8$

⑧ Parallel to u: $2\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$ Parallel to v: $2\begin{pmatrix} -5 \\ -2 \\ 4 \end{pmatrix}$

* multiply by <u>any</u> constant to stay parallel

(9) If $u \cdot v = 0$, then they are perpendicular. But back in #7, $u \cdot v = -8$, so they are <u>not</u> perpendicular.

(10) Parallel vectors are scalar multiples of each other.

$$k\begin{pmatrix}-5\\6\\-1\end{pmatrix} = \begin{pmatrix}10\\p\\q\end{pmatrix}$$

If you look at the top, k has to equal -2.

$$-2\begin{pmatrix}-5\\6\\-1\end{pmatrix} = \begin{pmatrix}10\\-12\\2\end{pmatrix}, \text{ so } \begin{matrix}p = -12\\ q = 2\end{matrix}$$

(11) $$\cos\theta = \frac{3(-2)-2(-4)+4(1)}{\sqrt{29}\cdot\sqrt{21}} = \frac{6}{\sqrt{29}\cdot\sqrt{21}}$$

$\theta = 75.9°$ or 1.33 radians

(12) $$\cos\theta = \frac{3(-2)-2(-4)+4(1)}{\sqrt{29}\cdot\sqrt{21}}$$

$\theta = 75.9°$ or 1.33 radians

(13)
$$-2+3t = 6-2p \xrightarrow{x-2} 4-6t = -12+4p$$
$$-3-2t = 5-4p \longrightarrow -3-2t = 5-4p$$
$$1-8t = -7$$
$$-8t = -8$$
$$t = 1$$

$$\begin{pmatrix}-2\\-3\\1\end{pmatrix} + 1\begin{pmatrix}3\\-2\\4\end{pmatrix} = \begin{pmatrix}1\\-5\\5\end{pmatrix}$$

$T = (1,-5,5)$

(14) Use $t=1$ from #13. time $= 1$

(15) $-14 = -2+3t$ so $y = -3+4(-2) = 5$

$-12 = 3t$

$-4 = t$

(16) $|a| = \sqrt{8^2+2^2+0.5^2} = \sqrt{68.25}$

$|b| = \sqrt{6^2+(-3)^2+(-.6)^2} = \sqrt{45.36}$

(17) Another way to ask for the intersection.

$5+8s = 13+6t \longrightarrow 5+8s = 13+6t$

$30+2s = 26-3t \xrightarrow{\times 2} 60+4s = 52-6t$

$65+12s = 65$

$12s = 0$

$s = 0$

$a = \begin{pmatrix} 5 \\ 30 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 8 \\ 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ 0 \end{pmatrix}$ Intersection $= (5, 30)$

(18) height of $a = 0$ for b, we have to solve for t. Using an equation from #17,

$5+8(0) = 13+6t$

$5 = 13+6t$

$-8 = 6t$

$-\frac{8}{6} = t$

$-4/3 = t$, so $z = 0 + {-4/3}(-.6)$

$z = .8$

(19) Already solved for:

$s = 0$, $t = -4/3$

(20) The airplanes will not collide, since they aren't at the intersection at the same time.