Key

Vectors Practice Packet #1

1. The vectors $\vec{i}$, $\vec{j}$ are unit vectors along the x-axis and y-axis respectively.
The vectors $\vec{u} = -\vec{i} + 2\vec{j}$ and $\vec{v} = 3\vec{i} + 5\vec{j}$ are given.

(a) Find $\vec{u} + 2\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$.

A vector $\vec{w}$ has the same direction as $\vec{u} + 2\vec{v}$, and has a magnitude of 26.

(b) Find $\vec{w}$ in terms of $\vec{i}$ and $\vec{j}$.

Working:

a) $\begin{pmatrix}-1\\2\end{pmatrix} + 2\begin{pmatrix}3\\5\end{pmatrix} = \begin{pmatrix}5\\12\end{pmatrix}$

b) $w = k\begin{pmatrix}5\\12\end{pmatrix} = \begin{pmatrix}5k\\12k\end{pmatrix}$

$\sqrt{(5k)^2 + (12k)^2} = 26$

$\sqrt{25k^2 + 144k^2} = 26$

$\sqrt{169k^2} = 26$

$13k = 26$

$k = 2$

Answers:

(a) $5i + 12j$

(b) $10i + 24j$

(Total 4 marks)

2. Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p.

(b) Given that $q|\mathbf{u}| = 14$, find the value of q.

a) $2(4) + 3(1) - 1(-p) = 0$ ← dot product $= 0$

$p = -11$

b) $|u| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$

$q\sqrt{14} = 14$

$q = \frac{14}{\sqrt{14}}$ or $\sqrt{14}$

(Total 6 marks)

$\overrightarrow{OA}$ $\overrightarrow{OB}$ $\overrightarrow{OC}$

3. Points A, B, and C have position vectors $4i + 2j$, $i - 3j$ and $-5i - 5j$. Let D be a point on the x-axis such that ABCD forms a parallelogram.

A(4,2) B(1,-3) C(-5,-5)

(a) (i) Find $\overrightarrow{BC}$. $\begin{pmatrix} -5-1 \\ -5+3 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$

(ii) Find the position vector of D.

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$
$= \overrightarrow{OA} + \overrightarrow{BC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

(4)

(b) Find the angle between $\overrightarrow{BD}$ and $\overrightarrow{AC}$.

$\overrightarrow{BD} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -9 \\ -7 \end{pmatrix}$

(6)

(Total 10 marks)

$\cos\theta = \dfrac{27 - 21}{\sqrt{18} \cdot \sqrt{130}}$

$\theta = 82.9°$ (3 sig. fig)

4. Find the angle between the following vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, giving your answer to the nearest degree.

$\boldsymbol{a} = -4i - 2j$
$\boldsymbol{b} = i - 7j$

Working:

$a = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ -7 \end{pmatrix}$

$\cos\theta = \dfrac{-4(1) - 2(-7)}{\sqrt{16+4} \cdot \sqrt{1+49}} = \dfrac{10}{\sqrt{20} \cdot \sqrt{50}}$

Answer:

$\theta = 72°$

(Total 4 marks)

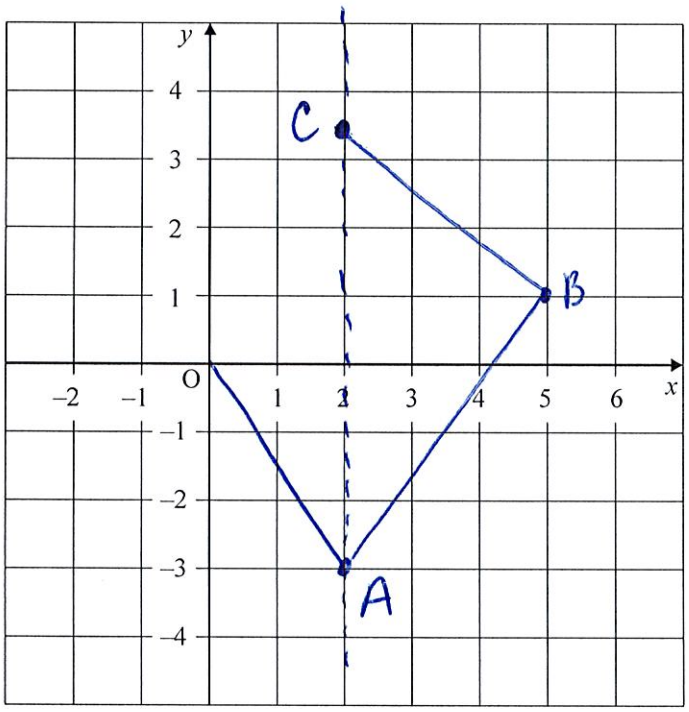
5. The triangle ABC is defined by the following information

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$, $\overrightarrow{AC}$ is parallel to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

So A (2, -3)

$\overrightarrow{AB} \perp \overrightarrow{BC}$

(a) On the grid below, draw an accurate diagram of triangle ABC.



C = (2, y)

B = (5, 1)

(b) Write down the vector $\overrightarrow{OC}$.

Working:

$\overrightarrow{BC} = \begin{pmatrix} 2-5 \\ y-1 \end{pmatrix} = \begin{pmatrix} -3 \\ y-1 \end{pmatrix}$

Since $\overrightarrow{AB} \perp \overrightarrow{BC}$,

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ y-1 \end{pmatrix} = 0$

$-9 + 4y - 4 = 0$

$y = 13/4$

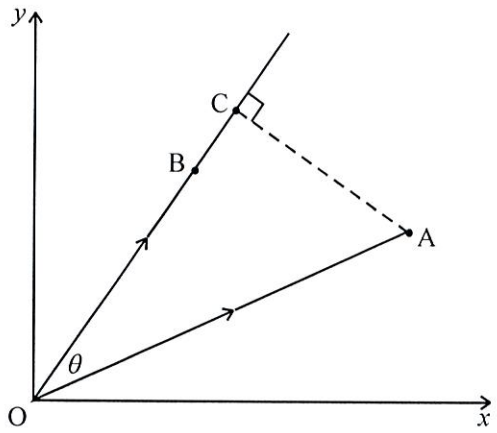
Answer:

(b) $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 13/4 \end{pmatrix}$

(Total 4 marks)

6. The following diagram shows the point O with coordinates (0, 0), the point A with position vector $\boldsymbol{a} = 12\boldsymbol{i} + 5\boldsymbol{j}$, and the point B with position vector $\boldsymbol{b} = 6\boldsymbol{i} + 8\boldsymbol{j}$. The angle between (OA) and (OB) is θ.

Diagram not to scale $|b| = \sqrt{6^2+8^2} = 10$



Find

(i) $|\boldsymbol{a}|$; $\sqrt{12^2+5^2} = 13$

(ii) a unit vector in the direction of $\boldsymbol{b}$; $\frac{1}{10}(6i+8j)$

(iii) the **exact** value of $\cos\theta$ in the form $\frac{p}{q}$, where, $p, q \in \mathbb{Z}$.

(not θ)

$$\cos\theta = \frac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{|OA||OB|} = \frac{12(6)+5(8)}{13\cdot 10} = \frac{112}{130}$$

(Total 6 marks)