Vectors Between Points, Parallel Vectors, and Angles

* End point − beginning point

Given the points $A(-1,2,5)$ and $B(3,1,-2)$, find

a) $\overrightarrow{AB} = \begin{pmatrix} 3-(-1) \\ 1-2 \\ -2-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ -7 \end{pmatrix}$

b) $\overrightarrow{BA} = \begin{pmatrix} -1-3 \\ 2-1 \\ 5-(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}$

Vectors that have the origin as the starting point are also called position vectors.

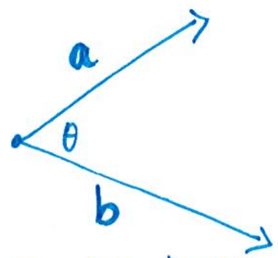
c) $\overrightarrow{OA} = \begin{pmatrix} -1-0 \\ 2-0 \\ 5-0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$

d) $\overrightarrow{OB} = \begin{pmatrix} 3-0 \\ 1-0 \\ -2-0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

* O is for origin : (0,0,0)

Angles between Vectors

$$\cos\theta = \frac{a_1b_1 + a_2b_2}{|a||b|} = \frac{a \cdot b}{|a||b|}$$



$a \cdot b = a_1b_1 + a_2b_2$

"dot product" = multiply corresponding components, then add.
= also called scalar product

* angles have vectors w/ same starting point.

Examples:

1) If $\mathbf{a} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, find $\mathbf{a} \bullet \mathbf{b}$, $\mathbf{b} \bullet \mathbf{a}$, and the angle between **a** & **b**.

$a \cdot b = -1(2) + 5(-4) = -2 + -20 = -22$
$b \cdot a = 2(-1) + -4(5) = -2 + -20 = -22$
} order doesn't matter!

Finding the angle

$\cos\theta = \dfrac{-1(2) + 5(-4)}{\sqrt{(-1)^2 + 5^2} \cdot \sqrt{2^2 + (-4)^2}}$ used formula

$\cos\theta = \dfrac{-22}{\sqrt{26} \cdot \sqrt{20}}$ simplified

$\theta = 165°$ (3 sig figs) used calculator to solve.

2) Find t so that $m = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ & $n = \begin{pmatrix} t \\ 2 \\ -6 \end{pmatrix}$ are a) parallel and b) perpendicular.

scalar multiples of each other

dot product = 0

Set corresponding components equal.

Parallel

$$K\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} t \\ 2 \\ -6 \end{pmatrix}$$

$2K = t \rightarrow 2(-2) = t$

$\boxed{-4 = t}$

$-K = 2$

$3K = -6$

$K = -2$

Perpendicular

$2(t) - 1(2) + 3(-6) = 0$

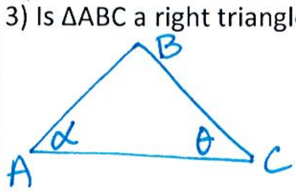
$2t - 2 - 18 = 0$

$2t - 20 = 0$

$2t = 20$

$\boxed{t = 10}$

3) Is ΔABC a right triangle? $A(5,1,2), B(6,-1,0), C(3,2,0)$ *Need to find a right angle.



θ is made w/ $\overrightarrow{CA}$ & $\overrightarrow{CB}$

$$\overrightarrow{CA} = \begin{pmatrix} 5-3 \\ 1-2 \\ 2-0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

$$\overrightarrow{CB} = \begin{pmatrix} 6-3 \\ -1-2 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}$$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = 2(3) + -1(-3) + 2(0) = 9$

So angle C is <u>not</u> a right angle.

α is made w/ $\overrightarrow{AB}$ & $\overrightarrow{AC}$

$$\overrightarrow{AB} = \begin{pmatrix} 6-5 \\ -1-1 \\ 0-2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 3-5 \\ 2-1 \\ 0-2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 1(-2) + -2(1) + -2(-2)$
$= 0$

When dot product is zero, we have a right angle, so α is $90°$.

4) Find the measure of the angle between $2x + y = 5$ and $3x - 2y = 8$.

Get y by itself: $y = -2x + 5$ $\quad y = \frac{3}{2}x - 4$

slope $= \frac{-2}{1}$ $\quad$ slope $= \frac{3}{2}$

Vectors: $\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$ $\quad$ direction: $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\quad$ direction: $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Slope: $\left(\frac{\Delta y}{\Delta x}\right)$ $\quad$ (flip the slope)

Use formula for angle: $\quad \cos\theta = \dfrac{1(2) + -2(3)}{\sqrt{1^2 + (-2)^2} \cdot \sqrt{2^2 + 3^2}}$

$\cos\theta = \dfrac{-4}{\sqrt{5} \cdot \sqrt{13}}$

Use degree mode this time.

$\theta = 119.7°$
(or $60.3°$)