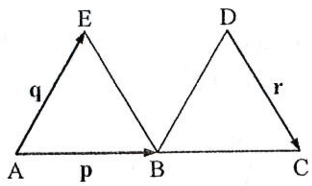
2 The figure alongside consists of two equilateral triangles. A, B, and C lie on a straight line.

$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AE} = \mathbf{q}$, and $\overrightarrow{DC} = \mathbf{r}$.

Which of the following statements are true?



a $\overrightarrow{EB} = \mathbf{r}$ T **b** $|\mathbf{p}| = |\mathbf{q}|$ T **c** $\overrightarrow{BC} = \mathbf{r}$ F

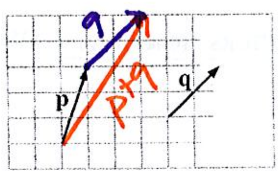
d $\overrightarrow{DB} = \mathbf{q}$ F **e** $\overrightarrow{ED} = \mathbf{p}$ T **f** $\mathbf{p} = \mathbf{q}$ F

$\overrightarrow{DB} = -\mathbf{q}$

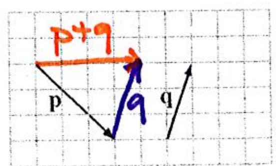
EXERCISE 12B.1

1 Use the given vectors $\mathbf{p}$ and $\mathbf{q}$ to construct $\mathbf{p} + \mathbf{q}$:

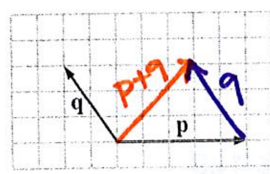
a



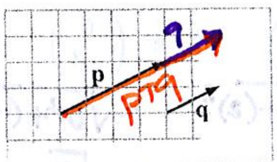
b



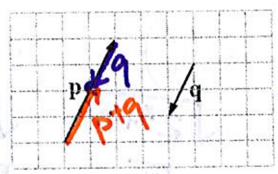
c



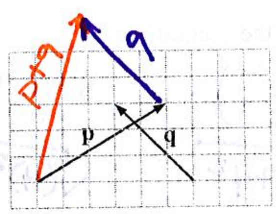
d



e



f



2 Find a single vector which is equal to:

a $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

b $\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$

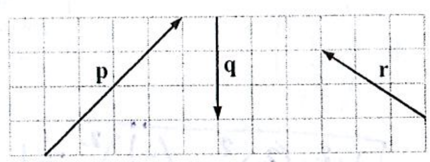
c $\overrightarrow{AB} + \overrightarrow{BA} = \mathbf{0}$

d $\underbrace{\overrightarrow{AB} + \overrightarrow{BC}} + \overrightarrow{CD}$
$= \overrightarrow{AC} + \overrightarrow{CD}$
$= \overrightarrow{AD}$

e $\underbrace{\overrightarrow{AC} + \overrightarrow{CB}} + \overrightarrow{BD}$
$= \overrightarrow{AB} + \overrightarrow{BD}$
$= \overrightarrow{AB}$

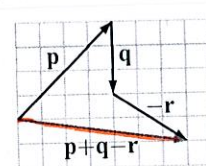
f $\underbrace{\overrightarrow{BC} + \overrightarrow{CA}} + \overrightarrow{AB}$
$= \overrightarrow{BA} + \overrightarrow{AB}$
$= \mathbf{0}$

2 For the vectors illustrated, show how to construct:

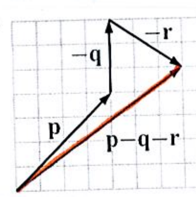


a $\mathbf{p} + \mathbf{q} - \mathbf{r}$ **b** $\mathbf{p} - \mathbf{q} - \mathbf{r}$ **c** $\mathbf{r} - \mathbf{q} - \mathbf{p}$

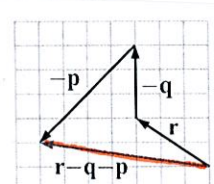
a



b



c



3 For points A, B, C, and D, simplify the following vector expressions:

a $\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$

b $\overrightarrow{AD} - \overrightarrow{BD} = \overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$

c $\overrightarrow{AC} + \overrightarrow{CA} = \mathbf{0}$

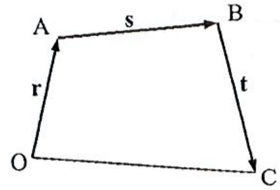
d $\underbrace{\overrightarrow{AB} + \overrightarrow{BC}} + \overrightarrow{CD}$
$= \overrightarrow{AC} + \overrightarrow{CD}$
$= \overrightarrow{AD}$

e $\underbrace{\overrightarrow{BA} - \overrightarrow{CA}} + \overrightarrow{CB}$
$= \underbrace{\overrightarrow{BA} + \overrightarrow{AC}} + \overrightarrow{CB}$
$= \overrightarrow{BC} + \overrightarrow{CB}$
$= \mathbf{0}$

f $\overrightarrow{AB} - \overrightarrow{CB} - \overrightarrow{DC}$
$= \underbrace{\overrightarrow{AB} + \overrightarrow{BC}} - \overrightarrow{DC}$
$= \overrightarrow{AC} + \overrightarrow{CD}$
$= \overrightarrow{AD}$

2 **a** Find, in terms of **r**, **s**, and **t**:

i $\overrightarrow{OB}$ **ii** $\overrightarrow{CA}$ **iii** $\overrightarrow{OC}$



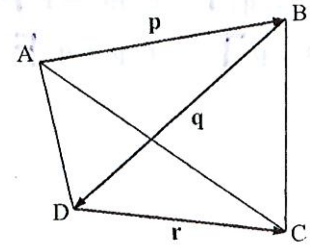
$\overrightarrow{OB} = r + s$

$\overrightarrow{CA} = -t - s$

$\overrightarrow{OC} = r + s + t$

b Find, in terms of **p**, **q**, and **r**:

i $\overrightarrow{AD}$ **ii** $\overrightarrow{BC}$ **iii** $\overrightarrow{AC}$



$\overrightarrow{AD} = p + q$

$\overrightarrow{BC} = q + r$

$\overrightarrow{AC} = p + q + r$

EXERCISE 12D

1 Find the magnitude of:

a $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ **b** $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ **c** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ **d** $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ **e** $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

a: $= \sqrt{3^2+4^2} = \sqrt{25} = 5$

b: $\sqrt{(-4)^2+(3)^2} = \sqrt{25} = 5$

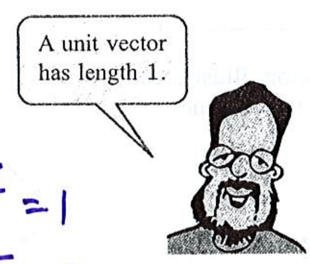
c: $\sqrt{2^2+0^2} = \sqrt{4} = 2$

d: $\sqrt{(-2)^2+(2)^2} = \sqrt{8}$

e: $= \sqrt{0^2+(-3)^2} = \sqrt{9} = 3$

14 Find k given the unit vector:

a $\begin{pmatrix} -\frac{1}{2} \\ k \\ \frac{1}{4} \end{pmatrix}$ **b** $\begin{pmatrix} k \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$



a:

$\sqrt{\left(-\frac{1}{2}\right)^2 + k^2 + \left(\frac{1}{4}\right)^2} = 1$

$\sqrt{k^2 + \frac{5}{16}} = 1$

$k^2 = \frac{11}{16}$

$k = \pm\frac{\sqrt{11}}{4}$

b:

$\sqrt{k^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2} = 1$

$\sqrt{k^2 + \frac{5}{9}} = 1$

$k^2 = \frac{4}{9}$

$k = \pm\frac{2}{3}$