Key

Vectors Practice Packet #2

1. In this question, the vector $\begin{pmatrix}1\\0\end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix}0\\1\end{pmatrix}$ km a displacement due north.

Two crews of workers are laying an underground cable in a north–south direction across a desert. At 06:00 each crew sets out from their base camp which is situated at the origin (0, 0). One crew is in a Toyundai vehicle and the other in a Chryssault vehicle.

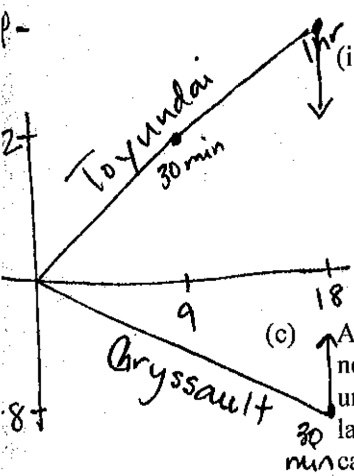
The Toyundai has velocity vector $\begin{pmatrix}18\\24\end{pmatrix}$ km h^{-1}, and the Chryssault has velocity vector $\begin{pmatrix}36\\-16\end{pmatrix}$ km h^{-1}.

$\sqrt{18^2+24^2} = 30$ $\qquad$ $\sqrt{36^2+16^2} = 39.4$

(a) Find the speed of each vehicle. **(2)**

(b) (i) Find the position vectors of each vehicle at 06:30. T(9, 12) $\quad$ C(18, −8)

(½ hour) **(2)**



(ii) Hence, or otherwise, find the distance between the vehicles at 06:30. **(3)**

$d = \sqrt{(9-18)^2+(12+8)^2} = \sqrt{481} = 21.9$ Km

(c) At this time (06:30) the Chryssault stops and its crew begin their day's work, laying cable in a northerly direction. The Toyundai continues travelling in the same direction at the same speed until it is exactly north of the Chryssault. The Toyundai crew then begin their day's work, laying cable in a southerly direction. At what time does the Toyundai crew begin laying cable?

7:00 am **(4)**

(d) Each crew lays an average of 800 m of cable in an hour. If they work non-stop until their lunch break at 11:30, what is the distance between them at this time?

C works for 5 hrs = 4 Km so @ (18, −4)

T works for 4.5 hrs = 3.6 Km, so @ (18, 20.4)

Distance = 20.4 − (−4) = 24.4 Km **(4)**

(e) How long would the Toyundai take to return to base camp from its lunch-time position, assuming it travelled in a straight line and with the same average speed as on the morning journey? (Give your answer to the nearest minute.)

T distance at lunch = $\sqrt{18^2+20.4^2} = 27.2$ Km **(5)**

(Total 20 marks)

$d = rt$

$\frac{27.2}{30} = \frac{30t}{30}$ $\qquad$ $t = 54$ minutes

2. The points A and B have the position vectors $\overset{A}{\begin{pmatrix} 2 \\ -2 \end{pmatrix}}$ and $\overset{B}{\begin{pmatrix} -3 \\ -1 \end{pmatrix}}$ respectively.

(a) (i) Find the vector $\overrightarrow{AB}$. $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$

(ii) Find $|\overrightarrow{AB}|$.

$\sqrt{5^2+1^2} = \sqrt{26}$

(4)

The point D has position vector $\begin{pmatrix} d \\ 23 \end{pmatrix}$.

(b) Find the vector $\overrightarrow{AD}$ in terms of d. $\begin{pmatrix} d-2 \\ 25 \end{pmatrix}$

(2)

The angle $B\hat{A}D$ is 90°. that means $\overrightarrow{AB} \perp \overrightarrow{AD}$ & $\overrightarrow{AB} \cdot \overrightarrow{AD} = 0$

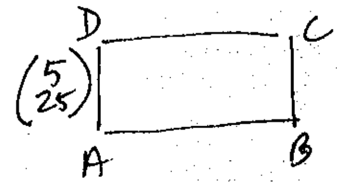
(c) (i) Show that $d = 7$. $-5(d-2) + 25(1) = 0$

$-5d + 10 + 25 = 0$

$d = 7$

(ii) Write down the position vector of the point D. $\begin{pmatrix} 7 \\ 23 \end{pmatrix}$

(3)



The quadrilateral ABCD is a rectangle.

(d) Find the position vector of the point C. $\begin{pmatrix} 2 \\ 24 \end{pmatrix}$

(4)

(e) Find the area of the rectangle ABCD.

$|\overrightarrow{AB}| \cdot |\overrightarrow{AD}| = \sqrt{26} \cdot \sqrt{650} = 130$

(2)
(Total 15 marks)

3. Let $v = 3i + 4j + k$ and $w = i + 2j - 3k$. The vector $v + pw$ is perpendicular to w. Find the value of p.

$(v + pw) \cdot w = 0$

$pw = pi + 2pj - 3pk$

$v + pw = (3+p)i + (4+2p)j + (1-3p)k$

$(v + pw) \cdot w = (3+p)(1) + (4+2p)(2) + (1-3p)(-3) = 0$

$3 + p + 8 + 4p - 3 + 9p = 0$

$14p = -8$

$p = \frac{-4}{7}$

(Total 7 marks)

4. (a) Find the scalar product of the vectors $\begin{pmatrix} 60 \\ 25 \end{pmatrix}$ and $\begin{pmatrix} -30 \\ 40 \end{pmatrix}$.

$60(-30) + 25(40) = -800$

(b) Two markers are at the points P (60, 25) and Q (–30, 40). A surveyor stands at O (0, 0) and looks at marker P. Find the angle she turns through to look at marker Q.

$\cos\theta = \dfrac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|\overrightarrow{OP}|\,|\overrightarrow{OQ}|} = \dfrac{-800}{\sqrt{4225} \cdot \sqrt{2500}} = \dfrac{-800}{65 \cdot 50}$

(Total 6 marks)

$\theta = 104°$ or 1.82 radians