① $6y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5y^4\frac{dy}{dx} = 4x^3 - 6x^2 + 2x$

$$\frac{dy}{dx}\frac{\cancel{(6y^2+2y-5y^4)}}{\cancel{6y^2+2y-5y^4}} = \frac{4x^3-6x^2+2x}{6y^2+2y-5y^4}$$

horizontal tangents when $4x^3-6x^2+2x=0$

$x = 0, \frac{1}{2}, 1$

vertical tangents when $y = -.856, -.379, 0, 1.235$

denominator = 0

② $\cos(x+y)\cdot(1+\frac{dy}{dx}) - \sin(x-y)(1-\frac{dy}{dx}) = 0$

$\cos(\pi)(1+\frac{dy}{dx}) - \sin(0)(1-\frac{dy}{dx}) = 0$

$-1 - \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -1$ $\qquad y - \frac{\pi}{2} = -1(x - \frac{\pi}{2})$

③ a. $y = a^x$

$\log_a y = x$

$\frac{\ln y}{\ln a} = x$

$\frac{1}{\ln a} \cdot \ln y = x$

b. $\frac{1}{\ln a} \cdot \frac{1}{y}\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \ln a \cdot y$

c. $\frac{dy}{dx} = \ln a \cdot y$

$\frac{dy}{dx} = \ln a \cdot a^x$

this is the derivative of $y = a^x$.

④ a. $6x^2 - 2y\frac{dy}{dx} = 3\frac{dy}{dx}$

$$-2y\frac{dy}{dx} - 3\frac{dy}{dx} = -6x^2$$

$$\frac{dy}{dx}(2y+3) = 6x^2$$

$$\frac{dy}{dx} = \frac{6x^2}{2y+3}$$

b. $3x^2 - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$

$$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$$

$$\frac{dy}{dx}(2y-x) = y-3x^2$$

$$\frac{dy}{dx} = \frac{y-3x^2}{2y-x}$$

c. $x^2\frac{dy}{dx} + y\cdot 2x + y^2\cdot 1 + x\cdot 2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx}\frac{(x^2+2xy)}{x^2+2xy} = \frac{-2xy-y^2}{x^2+2xy}$$

$$\frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}$$

d. $\sin x \cos y = \frac{1}{2}$

$(\sin x)(-\sin y)\frac{dy}{dx} + (\cos y)(\cos x) = 0$

$-\sin x \sin y \frac{dy}{dx} = -\cos x \cos y$

$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y} = \cot x \cot y$

⑤ $8x + 2y\frac{dy}{dx} - 8 + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y+4) = 8 - 8x$

$\frac{dy}{dx} = \frac{8-8x}{2y+4}$

horizontal tangents, set $\frac{dy}{dx} = 0$

$8 - 8x = 0$

$x = 1$, so

$(1,-4)(1,0)$

⑥ Set denominator from #5 (above) equal to zero.

$2y + 4 = 0$

$y = -2$, so $(0,-2),(2,-2)$

⑦ $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x^{-1/3}}{y^{-1/3}} = \frac{y^{1/3}}{x^{1/3}}$

$\left.\frac{dy}{dx}\right|_{(8,1)} = \frac{2}{1} = 2$

<u>tangent line</u>

$y - 1 = 2(x-8)$

<u>normal line</u>

$y - 1 = -\frac{1}{2}(x-8)$

⑧ $2y\frac{dy}{dx} = 2x+2$

$$\frac{dy}{dx} = \frac{2x+2}{2y} = \frac{x+1}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(1) - (x+1)\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - (x+1)\left(\frac{x+1}{y}\right)}{y^2} = \left(y - \frac{(x+1)^2}{y}\right) \cdot \frac{1}{y^2}$$

$$= \frac{1}{y} - \frac{(x+1)^2}{y^3} \text{ or } \frac{y^2 - (x+1)^2}{y^3}$$