Unit 2 Review

1) a. $f'(x) = \frac{(x+2)(1) - (x)(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$

b. (Hint, plug in -1 & a into f(x))

$$\frac{\frac{-1}{-1+2} - \frac{a}{a+2}}{-1-a} = \frac{\frac{-1}{1} - \frac{a}{a+2}}{-1-a} = \frac{\frac{-1(a+2) - a}{a+2}}{-1-a}$$

$$= \frac{-2a-2}{a+2} \cdot \frac{1}{-1-a} = \frac{2(-a-1)}{a+2} \cdot \frac{1}{-1-a} = \frac{2}{a+2}$$

c. $(1, \frac{1}{3})$ & $(2, \frac{1}{2})$

ave. rate of change (slope) $= \frac{\frac{1}{2} - \frac{1}{3}}{2-1} = \frac{\frac{1}{6}}{1} = \frac{1}{6}$

d. instantaneous = derivative

$f'(3) = \frac{2}{(3+2)^2} = \frac{2}{25}$

e. $f'(-1) = \frac{2}{(-1+2)^2} = 2$ $\quad f(1) = \frac{1}{3}$

$y - \frac{1}{3} = 2(x-1)$

f. normal = perpendicular to tangent

so $m = \frac{-1}{2}$ $\quad y - \frac{1}{3} = \frac{-1}{2}(x-1)$

g. $y - \frac{1}{3} = 2(1.1-1)$

$y - \frac{1}{3} = 2(.1)$

$y = .2 + \frac{1}{3} = \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$

h. Estimate is an over approximation since the graph of f is concave down at x=1, therefore a line tangent to f at x=1 would sit above the graph.

2. Need slope of tangent, so expand & rearrange.

$$x-\frac{1}{2}=-2y-8$$

$$2y=-x-\frac{15}{2}$$

$$y=\boxed{\frac{-1}{2}}x-\frac{15}{4}$$

$\to f'(-1)$

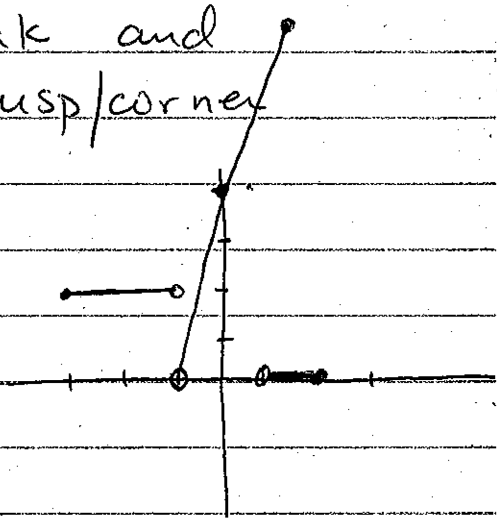
3. a. Discontinuous at $x=1$, a break

b. Not Differentiable at $x=1$, a break and $x=-2$, a cusp/corner

c. $f(x)=\begin{cases}2x-6, & -3\le x<-2\\ 2(x+1)^2, & -2\le x\le 1\\ 2, & 1<x\le 2\end{cases}$

d. $f'(x)=\begin{cases}2, & -3\le x<-2\\ 4x+4, & -2<x\le 1\\ 0, & 1<x\le 2\end{cases}$

e.



4. a. horizontal tangents are where $h'(x)=0$, so $x=-2, 3$

b. $(3,\infty)$ - increasing $(-\infty,-2)\cup(-2,3)$ - decreasing

c. concave up: $(-\infty,-2)\cup(1,\infty)$

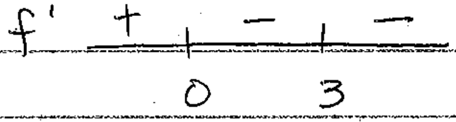
concave down: $(-2,1)$

5. a. f is increasing when $f'(x)>0$

inc: $(-\infty,0)$

f is decreasing when $f'(x)<0$

dec: $(0,3)\cup(3,\infty)$



#5 continued)

b. (Write line tangent at $x=1$)

$f(1)=3$ was given

$f'(1) = -2(4) = -8 \rightarrow y-3=-8(x-1)$

Plug in 1.1

$y-3=-8(1.1-1)$

$y-3=-8(.1)$

$y=3+-.8=2.2$

Under approximation since graph is concave up at $x=1$.

6. (slope) $\frac{-2-7}{-2-1} = \frac{-9}{-3} = 3 = f'(1)$

7. $p'(x)<0$ when $p(x)$ has negative slope.

so $(0,2) \cup (4,\infty)$