Trigonometric Identities & Equations

1) *If A is obtuse, then $\cos A$ is negative.*

$\sin^2 A + \cos^2 A = 1$

$\left(\frac{5}{13}\right)^2 + \cos^2 A = 1$

$\cos A = -\frac{12}{13}$

$\sin 2A = 2\sin A\cos A$

$= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$

$= \frac{-120}{169}$

2) a. $2\cos^2 x + \sin x =$

$2(1-\sin^2 x) + \sin x =$

$2 - 2\sin^2 x + \sin x$ (Can be in any order)

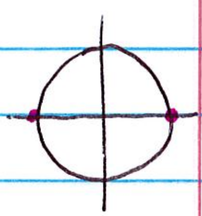
b. $2\cos^2 x + \sin x = 2$

$2 - 2\sin^2 x + \sin x = 2$

$-2\sin^2 x + \sin x = 0$

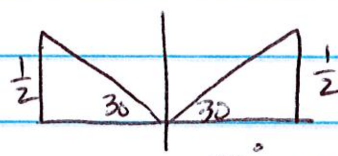
$\sin x - 2\sin^2 x = 0$

$\sin x(1 - 2\sin x) = 0$



$\sin x = 0$ $\quad$ $1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$



$30°, 150°$

$x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

3) If x is acute, then $\cos x$ is positive.

a) $\sin^2 x + \cos^2 x = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2 x = 1$

$\cos x = \frac{\sqrt{8}}{\sqrt{9}} = \frac{\sqrt{8}}{3}$

b) $\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{\sqrt{8}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$= \frac{8}{9} - \frac{1}{9}$

$= 7/9$

4) a. $2\sin^2 x - \cos x - 1 = 0$

$2(1-\cos^2 x) - \cos x - 1 = 0$

$2 - 2\cos^2 x - \cos x - 1 = 0$

$-2\cos^2 x - \cos x + 1 = 0$

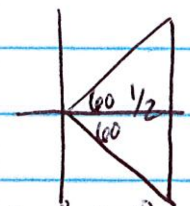
(or $2\cos^2 x + \cos x - 1 = 0$)

b. Using $2\cos^2 x + \cos x - 1 = 0$, let $w = \cos x$

$2w^2 + w - 1 = 0$

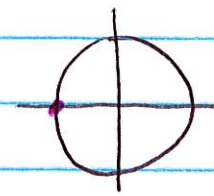
$(2w-1)(w+1) = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

c. 

$60^\circ, 300^\circ$

$\cos x = \frac{1}{2} \qquad \cos x = -1$



$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

5) $2\cos^2 x - \sin 2x = 0$

$2\cos^2 x - 2\sin x \cos x = 0$

$2\cos x(\cos x - \sin x) = 0$

$2\cos x = 0 \qquad \cos x - \sin x = 0$

$\cos x = 0 \qquad \cos x = \sin x$

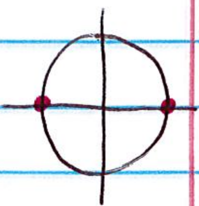
$x = \frac{\pi}{2}, \frac{\pi}{4}$. (double check your restriction)

6) $f(x) = 30\sin 3x \cos 3x$

a. We know $\sin 2\theta = 2\sin\theta\cos\theta$

let $\theta = 3x$: $\sin 6x = 2\sin 3x \cos 3x$

multiply both sides by 15: $15\sin 6x = 30\sin 3x \cos 3x$



b. $15\sin 6x = 0$

$\sin 6x = 0$

$\frac{6x}{6} = \frac{0, \pi, 2\pi}{6}$

$x = 0, \frac{\pi}{6}, \frac{\pi}{3}$

7) $3\cos x - 5\sin x = 0$ * Calculator Active *

Graph & solve! Window will need adjusting.

$x = 31°, 211°$

8) a. $2^2 + (BC)^2 = 3^2$ $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{5}}{3}$

$4 + (BC)^2 = 9$

$(BC)^2 = 5$

$BC = \sqrt{5}$

b. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right) = \frac{4\sqrt{5}}{9}$

c. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2 = \frac{4}{9} - \frac{5}{9} = \frac{-1}{9}$

9) a. $3\cos 2x + \sin x - 1 = 0$

$3(1 - 2\sin^2 x) + \sin x - 1 = 0$

$3 - 6\sin^2 x + \sin x - 1 = 0$

$-6\sin^2 x + \sin x + 2 = 0$ or $6\sin^2 x - \sin x - 2 = 0$

b. Let $w = \sin x$

$6w^2 - w - 2 = 0$

$(3w - 2)(2w + 1) = 0$

$(3\sin x - 2)(2\sin x + 1) = 0$

c. $\sin x = \frac{2}{3}$ $\sin x = -\frac{1}{2}$

there are 4 solutions

10) $3\sin^2 x = \cos^2 x$

$3\sin^2 x - \cos^2 x = 0$

$3\sin^2 x + (-1 + \sin^2 x) = 0$

$4\sin^2 x - 1 = 0$

$(2\sin x - 1)(2\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\quad\quad$ $\sin x = \frac{-1}{2}$ ← not possible for restriction given

$x = 30°, 150°$