correct substitution (A1)
eg $\quad 0.3 \times 0.6$
$\mathrm{P}(A \cap B)=0.18 \quad$ A1 $\quad N 2$
[2 marks]
20b. [2 marks]
correct substitution (A1)
eg $\mathrm{P}(A \cup B)=0.3+0.6-0.18$
$\mathrm{P}(A \cup B)=0.72 \quad A 1 \quad N 2$
[2 marks]
20c. [1 mark]


20d. [2 marks]
Markscheme
appropriate approach (M1)
eg $\quad 0.3-0.18, \mathrm{P}(A) \times \mathrm{P}\left(B^{\prime}\right)$
$\mathrm{P}\left(A \cap B^{\prime}\right)=0.12$ (may be seen in Venn diagram) A1 N2
[2 marks]

21a. [2 marks]
attempt to find number who took less than 45 minutes (M1)
eg line on graph (vertical at approx 45, or horizontal at approx 70)
70 students (accept 69) A1 N2
[2 marks]
21b. [3 marks]
55 students completed task in less than 35 minutes (A1)
subtracting their values (M1)
eg 70-55
15 students A1 N2
[3 marks]
21c. [2 marks]
correct approach (A1)
eg line from $y$-axis on 50
$k=33 \quad$ A1 $\quad N 2$
[2 marks]
22a. [2 marks]
Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with their working and check carefully for $\boldsymbol{F T}$.
attempt to standardize (M1)
eg $\quad z=\frac{21.8-20}{1.25}, 1.44$
$\mathrm{P}(T<21.8)=0.925 \quad$ A1 $\quad N 2$

## [2 marks]

22b. [5 marks]
Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with their working and check carefully for $\boldsymbol{F T}$.
attempt to subtract probabilities (M1)
eg $\mathrm{P}(T<21.8)-\mathrm{P}(T<k)=0.3,0.925-0.3$
$\mathrm{P}(T<k)=0.625 \quad$ A1

## EITHER

finding the $z$-value for 0.625 (A1)
eg $\quad z=0.3186$ (from tables), $z=0.3188$
attempt to set up equation using their $z$-value (M1)
eg $\quad 0.3186=\frac{k-20}{1.25},-0.524 \times 1.25=k-20$
$k=20.4 \quad$ A1 $\quad$ N3

OR
$k=20.4 \quad$ A3 $\quad$ N3
[5 marks]
23a. [5 marks]
(i) attempt to find $\mathrm{P}($ red $) \times \mathrm{P}($ red $) \quad$ (M1)
eg $\frac{3}{8} \times \frac{2}{7}, \frac{3}{8} \times \frac{3}{8}, \frac{3}{8} \times \frac{2}{8}$
$\mathrm{P}($ none green $)=\frac{6}{56}\left(=\frac{3}{28}\right) \quad$ A1 $\quad N 2$
(ii) attempt to find $\mathrm{P}($ red $) \times \mathrm{P}($ green $) \quad$ (M1)
eg $\frac{5}{8} \times \frac{3}{7}, \frac{3}{8} \times \frac{5}{8}, \frac{15}{56}$
recognizing two ways to get one red, one green (M1)
eg $2 \mathrm{P}(R) \times \mathrm{P}(G), \frac{5}{8} \times \frac{3}{7}+\frac{3}{8} \times \frac{5}{7}, \frac{3}{8} \times \frac{5}{8} \times 2$
$\mathrm{P}($ exactly one green $)=\frac{30}{56}\left(=\frac{15}{28}\right) \quad$ A1 $\quad N 2$

## [5 marks]

23b. [3 marks]
$\mathrm{P}($ both green $)=\frac{20}{56}$ (seen anywhere) $\quad$ (A1)
correct substitution into formula for $\mathrm{E}(X) \quad$ A1
$e g 0 \times \frac{6}{56}+1 \times \frac{30}{56}+2 \times \frac{20}{56}, \frac{30}{64}+\frac{50}{64}$
expected number of green marbles is $\frac{70}{56}\left(=\frac{5}{4}\right)$ A1 $\quad N 2$
[3 marks]
23c. [2 marks]
(i) $\mathrm{P}($ jar B$)=\frac{4}{6}\left(=\frac{2}{3}\right) \quad$ A1 N1
(ii) $\mathrm{P}($ red $\mid$ jar B$)=\frac{6}{8}\left(=\frac{3}{4}\right) \quad$ A1 $\quad$ N1
[2 marks]
23d. [6 marks]
recognizing conditional probability (M1)
$e g \mathrm{P}(A \mid R), \frac{\mathrm{P}(\mathrm{jar} \mathrm{A} \text { and red })}{\mathrm{P}(\mathrm{red})}$, tree diagram
attempt to multiply along either branch (may be seen on diagram) (M1)
$e g \mathrm{P}(\mathrm{jar} \mathrm{A}$ and red $)=\frac{1}{3} \times \frac{3}{8}\left(=\frac{1}{8}\right)$
attempt to multiply along other branch (M1)
$e g \mathrm{P}($ jar B and red $)=\frac{2}{3} \times \frac{6}{8}\left(=\frac{1}{2}\right)$
adding the probabilities of two mutually exclusive paths
${ }_{e g} \mathrm{P}(\mathrm{red})=\frac{1}{3} \times \frac{3}{8}+\frac{2}{3} \times \frac{6}{8}$
correct substitution

$$
\mathrm{P}(\text { jar } \mathrm{A} \mid \text { red })=\frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8}+\frac{2}{3} \times \frac{6}{8}}, \frac{\frac{1}{8}}{\frac{5}{8}}
$$

$\mathrm{P}($ jar $\mathrm{A} \mid$ red $)=\frac{1}{5}$
A1 N3
[6 marks]
24a. [2 marks]
valid approach (M1)
eg $35-26,26+p=36$
$p=9 \quad$ A1 $\quad N 2$
[2 marks]
24b. [4 marks]
(i) mean $=26.7 \quad$ A2 $\quad N 2$
(ii) recognizing that variance is (sd) ${ }^{2} \quad$ (M1)
eg 11.021... ${ }^{2}, \sigma=\sqrt{\operatorname{var}}, 11.158 \ldots{ }^{2}$
$\sigma^{2}=121 \quad A 1 \quad N 2$

## [4 marks]

25a. [2 marks]
evidence of valid approach (M1)
e.g. $92+52$, line on graph at $x=31$
$p=144 \quad A 1 \quad N 2$
[2 marks]

25b. [5 marks]
(i) evidence of valid approach (M1)
e.g. line on graph, $0.8 \times 160$, using complement
$=29.5 \quad$ A1 $\quad \mathrm{N} 2$
(ii) $Q_{1}=23$; $Q_{3}=29 \quad$ (A1)(A1)
$\mathrm{IQR}=6$ (accept any notation that suggests an interval) A1 N3 [5 marks]

26a. [1 mark]
median $=174(\mathrm{~cm}) \quad$ A1 $\quad N 1$
[1 mark]
26b. [2 marks]
attempt to find number shorter than 161 (M1)
e.g. line on graph, 12 boys
$p=\frac{12}{200}(=0.06) \quad$ A1 $\quad N 2$
[2 marks]

26c. [3 marks]

## METHOD 1

$18 \%$ have a height less than $h \quad$ (A1)
$0.18 \times 200=36$ (36 may be seen as a line on the graph) (A1)
$h=166(\mathrm{~cm}) \quad A 1 \quad N 2$
METHOD 2
$0.82 \times 200=164$ (164 may be seen as a line on the graph) (A1)
$200-164=36 \quad$ (A1)
$h=166(\mathrm{~cm}) \quad A 1 \quad N 2$
[3 marks]
27a. [1 mark]
$\mathrm{P}(X=2)=\frac{4}{14}\left(=\frac{2}{7}\right)_{\text {A1 }} \quad N 1$
[1 mark]
27b. [4 marks]
$\mathrm{P}(X=1)=\frac{1}{14}$
$\mathrm{P}(X=k)=\frac{k^{2}}{14}$
setting the sum of probabilities $=1 \quad$ M1
e.g. $\frac{1}{14}+\frac{4}{14}+\frac{k^{2}}{14}=1,5+k^{2}=14$
$k^{2}=9\left(\right.$ accept $\left.\frac{k^{2}}{14}=\frac{9}{14}\right) \quad$ A1
$k=3 \quad A G \quad N O$
[4 marks]

27c. [2 marks]
correct substitution into $\mathrm{E}(X)=\sum x \mathrm{P}(X=x) \quad$ a1
e.g. $1\left(\frac{1}{14}\right)+2\left(\frac{4}{14}\right)+3\left(\frac{9}{14}\right)$
$\mathrm{E}(X)=\frac{36}{14}\left(=\frac{18}{7}\right) \quad$ A1 $\quad N 1$

## [2 marks]

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