A QUADRATIC EQUATIONS

Acme Leather Jacket Co. makes and sells x leather jackets each week and their profit function is given by
$P = -12.5x^2 + 550x - 2125$ dollars.

How many jackets must be made and sold each week in order to obtain a weekly profit of \$3000?

Clearly we need to solve the equation:

$$-12.5x^2 + 550x - 2125 = 3000$$

We can rearrange the equation to give

$$12.5x^2 - 550x + 5125 = 0,$$

which is of the form $ax^2 + bx + c = 0$ and is thus a quadratic equation.

SOLVING QUADRATIC EQUATIONS

To solve quadratic equations we have the following methods to choose from:

- **factorise** the quadratic and use the **Null Factor law**:

  If $ab = 0$ then $a = 0$ or $b = 0$.

- **complete the square**
- use the **quadratic formula**
- use **technology**.

The **roots** or **solutions** of $ax^2 + bx + c = 0$ are the values of x which satisfy the equation, or make it true.

For example, consider $x^2 - 3x + 2 = 0$.

When $x = 2$, $x^2 - 3x + 2 = (2)^2 - 3(2) + 2$
$= 4 - 6 + 2$
$= 0$ ✓

So, $x = 2$ is a root of the equation $x^2 - 3x + 2 = 0$.

SOLVING BY FACTORISATION

Step 1: If necessary, rearrange the equation so one side is zero.
Step 2: Fully factorise the other side.
Step 3: Use the Null Factor law: If $ab = 0$ then $a = 0$ or $b = 0$.
Step 4: Solve the resulting linear equations.

Caution: Do not be tempted to divide both sides by an expression involving x.
If you do this then you may lose one of the solutions.

For example, consider $x^2 = 5x$.

Correct solution

$x^2 = 5x$
$\therefore\ x^2 - 5x = 0$
$\therefore\ x(x - 5) = 0$
$\therefore\ x = 0$ or 5

Incorrect solution

$x^2 = 5x$
$\therefore\ \frac{x^2}{x} = \frac{5x}{x}$ ← By dividing both sides by x, we lose the solution $x = 0$.
$\therefore\ x = 5$

Example 1 — Self Tutor

Solve for x: **a** $3x^2 + 5x = 0$ **b** $x^2 = 5x + 6$

a
$3x^2 + 5x = 0$
$\therefore x(3x + 5) = 0$
$\therefore x = 0$ or $3x + 5 = 0$
$\therefore x = 0$ or $x = -\frac{5}{3}$

b
$x^2 = 5x + 6$
$\therefore x^2 - 5x - 6 = 0$
$\therefore (x - 6)(x + 1) = 0$
$\therefore x = 6$ or -1

Example 2 — Self Tutor

Solve for x: **a** $4x^2 + 1 = 4x$ **b** $6x^2 = 11x + 10$

a
$4x^2 + 1 = 4x$
$\therefore 4x^2 - 4x + 1 = 0$
$\therefore (2x - 1)^2 = 0$
$\therefore x = \frac{1}{2}$

b
$6x^2 = 11x + 10$
$\therefore 6x^2 - 11x - 10 = 0$
$\therefore (2x - 5)(3x + 2) = 0$
$\therefore x = \frac{5}{2}$ or $-\frac{2}{3}$

Example 3 — Self Tutor

Solve for x: $3x + \frac{2}{x} = -7$

$3x + \frac{2}{x} = -7$
$\therefore x\left(3x + \frac{2}{x}\right) = -7x$ {multiplying both sides by x}
$\therefore 3x^2 + 2 = -7x$ {expanding the brackets}
$\therefore 3x^2 + 7x + 2 = 0$ {making the RHS 0}
$\therefore (x + 2)(3x + 1) = 0$ {factorising}
$\therefore x = -2$ or $-\frac{1}{3}$

EXERCISE 1A.1

1 Solve the following by factorisation:

a $4x^2 + 7x = 0$ **b** $6x^2 + 2x = 0$ **c** $3x^2 - 7x = 0$
d $2x^2 - 11x = 0$ **e** $3x^2 = 8x$ **f** $9x = 6x^2$
g $x^2 - 5x + 6 = 0$ **h** $x^2 = 2x + 8$ **i** $x^2 + 21 = 10x$
j $9 + x^2 = 6x$ **k** $x^2 + x = 12$ **l** $x^2 + 8x = 33$

2 Solve the following by factorisation:

a $9x^2 - 12x + 4 = 0$ **b** $2x^2 - 13x - 7 = 0$ **c** $3x^2 = 16x + 12$
d $3x^2 + 5x = 2$ **e** $2x^2 + 3 = 5x$ **f** $3x^2 + 8x + 4 = 0$
g $3x^2 = 10x + 8$ **h** $4x^2 + 4x = 3$ **i** $4x^2 = 11x + 3$
j $12x^2 = 11x + 15$ **k** $7x^2 + 6x = 1$ **l** $15x^2 + 2x = 56$

3 Solve for x:

a $(x + 1)^2 = 2x^2 - 5x + 11$ **b** $(x + 2)(1 - x) = -4$
c $5 - 4x^2 = 3(2x + 1) + 2$ **d** $x + \frac{2}{x} = 3$
e $2x - \frac{1}{x} = -1$ **f** $\frac{x + 3}{1 - x} = -\frac{9}{x}$