Key

NON-CALCULATOR

IB Math SL 2/ AB Calculus AB Midterm Review

1. If $y=(x^3+1)^2$, then $\frac{dy}{dx}=$

(A) $(3x^2)^2$ (B) $2(x^3+1)$ (C) $2(3x^2+1)$ (D) $3x^2(x^3+1)$ (E) $6x^2(x^3+1)$

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2. For $x \geq 0$, the horizontal line $y=2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0)=2$ (B) $\lim_{x\to\infty} f(x)=2$ (C) $f(2)$ is undefined (D) $\lim_{x\to 2} f(x)=\infty$ (E) $f(x)\neq 2$ for all $x \geq 0$

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3. If $y=\frac{2x+3}{3x+2}$, then $\frac{dy}{dx}=$

(A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (D) $\frac{-5}{(3x+2)^2}$ (E) $\frac{2}{3}$

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4. $\lim_{x\to\infty}\frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1

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5. If $f(x)=x\sqrt{2x-3}$, then $f'(x)=$

(A) $\frac{3x-3}{\sqrt{2x-3}}$ (B) $\frac{x}{\sqrt{2x-3}}$ (C) $\frac{1}{\sqrt{2x-3}}$ (D) $\frac{3-x}{\sqrt{2x-3}}$ (E) $\frac{5x-6}{2\sqrt{2x-3}}$

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6. If $f(x)=\ln(x+4+e^{-3x})$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

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5) $x\cdot\frac{1}{2}(2x-3)^{-1/2}(2)+(2x-3)^{1/2}$

$(2x-3)^{-1/2}[x+2x-3]$

$\frac{3x-3}{\sqrt{2x-3}}$

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7. $\lim_{x\to 2}\frac{e^{x^2}-e^4}{x-2}=$

(A) dne (B) e^4 (C) $2e^4$ (D) 0 (E) $4e^4$

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8. What is the slope of the tangent to the curve $3y^2-2x^2=6-2xy$ at the point (3, 2)?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

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9. $\frac{d}{dx}\cos^2(x^3)=$

(A) $6x^2\sin(x^3)\cos(x^3)$ (B) $6x^2\cos(x^3)$
(C) $\sin^2(x^3)$ (D) $-6x^2\sin(x^3)\cos(x^3)$ (E) $-2\sin(x^3)\cos(x^3)$

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10. Let f be the function with ***derivative*** given by $f'(x)=x^2-\frac{8}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty,-1)$ (B) $(-1,0)$ (C) $(-\infty,0)$ (D) $(0,2)$ (E) $(2,\infty)$

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11. Find $f'(x)$ if $f(x)=\tan x+\sin x$.

(A) $\sec^2 x+\cos x$ (B) $\sec^2 x-\cos x$
(C) $\sec x\tan x+\cos x$ (D) $\cot x+\cos x$ (E) $\sec x\tan x-\cos x$

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12. Let f be the function given by $f(x)=2xe^x$. The graph of f is concave down when

(A) $x<-2$ (B) $x>-2$ (C) $x<-1$ (D) $x>-1$ (E) $x<0$

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13. Find $\lim_{x\to 2}(3x^2+5)$

(A) 41 (B) 17 (C) 11 (D) 0 (E) 2

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7) $\lim_{x\to 2}\frac{2xe^{x^2}}{1}=4e^4$

9) $[\cos(x^3)]^2$

$2\cos(x^3)\cdot 3x^2\cdot\sin(x^3)$

10) $\frac{x^3-8}{x}$ $\quad\frac{+\ -\ +}{0\quad 2}$

11) $\sec^2 x+\cos x$

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14. Let $f(x)=\begin{cases} x+2 & \text{if } x\le 3 \\ 4x-7 & \text{if } x>3 \end{cases}$. Which of the following statements are true about f?

I. $\lim_{x\to 3} f(x)$ exists II. f is continuous at $x=3$ III. f is differentiable at $x=3$

(A) None (B) I only (C) II only (D) I & II only (E) I, II, & III

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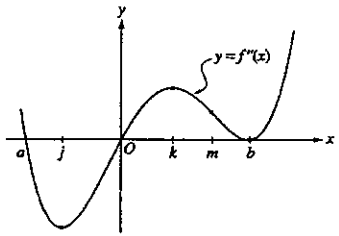
15. The second derivative of the function f is given by $f''(x)=x(x-a)(x-b)^2$. The graph of f'' is shown below. For what values of x does the graph of f have a point of inflection?



(A) 0 & a only (B) 0 & m only (C) b & j only (D) 0, a, & b (E) b, j, & k

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16. Let f be the function defined by $f(x)=4x^3-5x+3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x=-1$?

(A) $y=7x-3$ (B) $y=7x+7$ (C) $y=7x+11$ (D) $y=-5x-1$ (E) $y=-5x-5$

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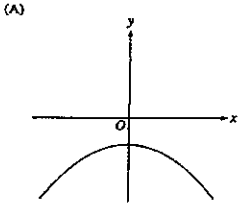
17. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are **negative** for all real values x. Which of the following could be the graph of f?

(A)



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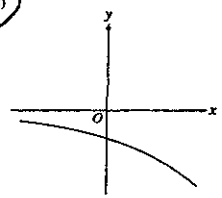

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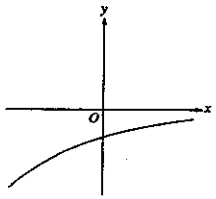

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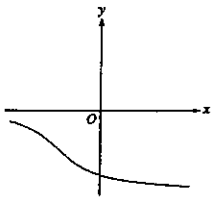
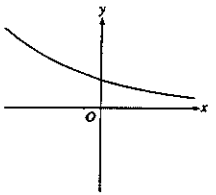


(E)



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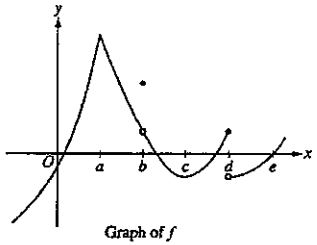
18. Given that $f(x)=x^3$, find the average rate of change of $f(x)$ on the interval [1, 3]

(A) 6 (B) 13 (C) $\sqrt{13}$ (D) $\frac{27}{2}$ (E) $\sqrt{\frac{13}{3}}$

(slope)

(1, 1) (3, 27)

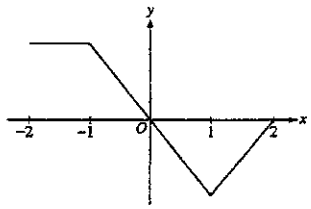
$$\frac{27-1}{3-1}=\frac{26}{2}=13$$



Graph of f

19. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e



Graph of f'

20. The graph of f', the derivative of f, is shown above. Which of the following statements is true about f?

(A) f is decreasing for $-1 \le x \le 1$ (B) f is increasing for $-2 \le x \le 0$ (C) f is increasing for $1 \le x \le 2$
(D) f has a local minimum at $x = 0$ (E) f is not differentiable at $x = -1$ and $x = 1$

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21. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$.
(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.
(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$.
(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.
(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

$\frac{4-(-5)}{1-(-2)} = \frac{9}{3} = 3$

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22. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

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(A) $0.04\pi \frac{m^2}{sec}$ (B) $0.4\pi \frac{m^2}{sec}$ (C) $4\pi \frac{m^2}{sec}$ (D) $20\pi \frac{m^2}{sec}$ (E) $100\pi \frac{m^2}{sec}$

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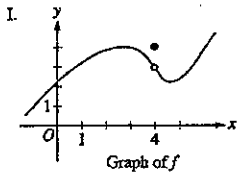
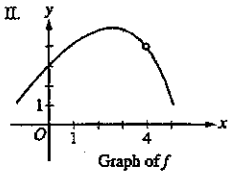
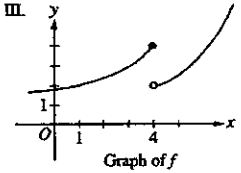
23. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I.


Graph of f

II.


Graph of f

III.


Graph of f

(A) I only (B) II only (C) III only (D) I & II only (E) I & III only

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24. If $f(x) = \begin{cases} \ln x & ; 0 < x \le 2 \\ x^2 \ln 2 & ; 2 < x \le 4 \end{cases}$, at what value of x does f attain its maximum value on the interval $[0, 2]$?

(A) ln 2 (B) ln 8 (C) ln 16 (D) 4 (E) nonexistent

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25. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

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26. Let f be the function with a first derivative of $f'(x) = \sin(x^3)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the interval $[0, 2]$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

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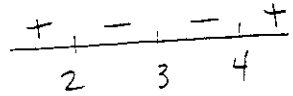
27. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$.
II. A relative minimum at $x = 3$.
III. A relative maximum at $x = 4$.



(A) I only (B) III only (C) I & III (D) II & III (E) I, II, & III
