IB Math SL Midterm Review #2 Name Key

Do your work on a separate sheet of paper. ***All questions are non-calculator except #1, 2, & 7.***

1. Find the term in x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

$\binom{5}{r}(3x^2)^{5-r}\left(-\frac{2}{x}\right)^r$

$\binom{5}{r}3^{5-r}\cdot(-2)^r x^{10-3r} = \boxed{1080x^4}$

$10-3r=4$
$-3r=-6$
$r=2$

(Total 6 marks)

2. Determine the constant term in the expansion of $\left(x - \frac{2}{x^2}\right)^9$.

$\binom{9}{r}x^{9-r}\left(-\frac{2}{x^2}\right)^r$

$\binom{9}{r}(-2)^r\cdot x^{9-3r} = \binom{9}{3}(-2)^3 = \boxed{-672}$

$9-3r=0$
$r=3$

(Total 4 marks)

3. The quadratic equation $4x^2 + 4kx + 9 = 0$, $\boxed{k > 0}$ has exactly one solution for x. Find the value of k.

↳ discriminant = 0

$(4k)^2 - 4(4)(9) = 0$
$16k^2 - 16(9) = 0$
$16(k^2 - 9) = 0$

$(k-3)(k+3) = 0$
$k = \pm 3$, but $k > 0$,
so $\boxed{k = 3}$

(Total 4 marks)

4. The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

discriminant is greater than zero.

$(-2k)^2 - 4(1)(1) > 0$
$4k^2 - 4 > 0$
$4(k^2 - 1) > 0$
$4(k-1)(k+1) > 0$

$+ \quad -1 \quad - \quad 1 \quad +$

so $\boxed{k < -1 \text{ and } k > 1}$

(Total 6 marks)

5. Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x-p)^2 + q$, where $a, p, q \in \mathbb{Z}$. $f(x) = 2(x-2)^2 - 3$

(b) Find the minimum value of $f(x)$.

vertex

$x = \frac{8}{4} = 2$ so $y = 2(2)^2 - 8(2) + 5 = -3$

$\boxed{(2, -3)}$

(Total 6 marks)

6. Find the sum of the infinite geometric series

$$\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \ldots$$

$r = \dfrac{-\frac{4}{9}}{\frac{2}{3}} = -\frac{2}{3}$

$S_\infty = \dfrac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)} = \dfrac{\frac{2}{3}}{\frac{5}{3}} = \boxed{\frac{2}{5}}$

(Total 4 marks)

7. (a) Consider the geometric sequence −3, 6, −12, 24, ….

(i) Write down the common ratio. $r = \frac{6}{-3} = -2$

(ii) Find the 15th term. $u_{15} = -3(-2)^{14} = -49152$

Consider the sequence $x - 3$, $x + 1$, $2x + 8$, ….

(3)

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms. $2, 6, 18$

(ii) Find the common ratio. $r = 3$

(2)

(c) Find the other value of x for which the sequence is geometric.

$$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$$

$$x^2 + 2x + 1 = 2x^2 + 2x - 24$$

$$0 = x^2 - 25$$

$x = \pm 5$, so $\boxed{x = -5 \text{ is other value}}$

(4)

(d) For this value of x, find $-8, -4, -2$

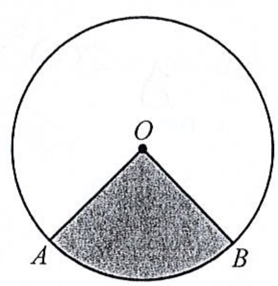
(i) the common ratio; $r = \frac{1}{2}$

(ii) the sum of the infinite sequence.

$$S_\infty = \frac{-8}{1-\frac{1}{2}} = \frac{-8}{\frac{1}{2}} = \boxed{-16}$$

(3)
(Total 12 marks)

8. O is the centre of the circle which has a radius of 5.4 cm.



The area of the shaded sector OAB is 21.6 cm^2. Find the length of the minor arc AB.

$21.6 = \frac{1}{2}\theta(5.4)^2$

$\theta = 1.48$

$l = \theta r$

$l = (1.48)(5.4)$

$l = 8$ (3 sig figs)

(Total 4 marks)

9. A triangle has sides of length 4, 5, 7 units. Find, to the nearest tenth of a degree, the size of the largest angle.

$7^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$

$\theta = 101.5^\circ$

(Total 4 marks)

10. The following diagram shows a circle of centre O, and radius 15 cm. The arc ACB subtends an angle of 2 radians at the centre O.

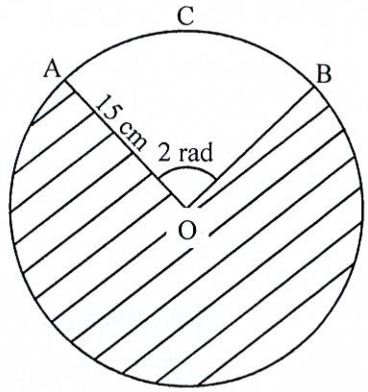


Diagram not to scale

$\hat{AOB} = 2$ radians
OA = 15 cm

Find

(a) the length of the arc ACB; $l = 2(15) = 30$

(b) the area of the shaded region.

$A = \pi(15)^2 - \frac{1}{2}(2)(15)^2 = 482$

(Total 6 marks)

11. In a triangle ABC, AB = 4 cm, AC = 3 cm and the area of the triangle is 4.5 cm^2.

Find the **two** possible values of the angle $\hat{BAC}$.

$4.5 = \frac{1}{2}(4)(3)\sin\theta$

$9 = 12\sin\theta$

$\frac{9}{12} = \sin\theta$

$\theta = 48.6^\circ, 131^\circ$

(Total 6 marks)