Arithmetic Sequences & Series

* $a = u$ ... they are interchangable

① a. $a_1 = 7$
$d = 2.5$
$n = 41$
$a_{41} = 7 + (41-1)(2.5)$
$\boxed{a_{41} = 107}$

b. $S_n = \frac{n}{2}(2a_1 + (n-1)d)$
$S_{101} = \frac{101}{2}(2(7) + (101-1)(2.5))$
$\boxed{S_{101} = 13{,}332}$

② $a_1 = 5$
$a_4 = 40$
$a_2 = ?$

$a_n = a_1 + (n-1)d$
$40 = 5 + (4-1)d$
$35 = 3d$
$\frac{35}{3} = d$

$a_2 = 5 + (2-1)\frac{35}{3}$
$\boxed{a_2 = \frac{50}{3}}$

③ $S_{40} = \frac{40}{2}(a_1 + a_{40})$
$1900 = 20(a_1 + 106)$
$95 = a_1 + 106$
$\boxed{-11 = a_1}$ $(a_1 = u_1)$

$a_{40} = -11 + (40-1)d$
$106 = -11 + 39d$
$117 = 39d$
$\boxed{3 = d}$

④ a. $u_1 = 5 + 2(1) = 7$
$u_2 = 5 + 2(2) = 9$
so $\boxed{d = 2}$

b. i) $115 = 7 + (n-1)(2)$
$108 = 2n - 2$
$110 = 2n$
$\boxed{55 = n}$

ii) $S_{55} = \frac{55}{2}(7 + 115)$
$= 3355$

⑤ $a_1 = -7$, $S_{20} = 620$

a. $S_{20} = \frac{20}{2}(2(-7) + (20-1)d)$
$620 = 10(-14 + 19d)$
$d = 4$

b. $a_{78} = -7 + (78-1)(4)$
$a_{78} = 301$
(or $u_{78} = 301$)

(6) Need to know how many terms there are (n).

$417 = 17 + (n-1)(10)$

$417 = 17 + 10n - 10$

$410 = 10n$

$41 = n$

$S_{41} = \frac{41}{2}(17 + 417) = \boxed{8897}$

(7) a. $64 = 7 + (20-1)d$

$64 = 7 + 19d$

$57 = 19d$

$\boxed{3 = d}$

b. $3709 = 7 + (n-1)(3)$

$3702 = 3n - 3$

$3705 = 3n$

$\boxed{1235 = n}$

(8) $u_1 = 32$, $u_2 = 16$, $u_3 = 8$ * Be sure to use geometric formulas *

a. $16 = 32r^{2-1}$

$\boxed{\frac{1}{2}} = r$

b. $u_6 = 32\left(\frac{1}{2}\right)^{6-1}$

$u_6 = 32\left(\frac{1}{2}\right)^5$

$u_6 = 32 \cdot \frac{1}{32} = \boxed{1}$

c. $S_\infty = \frac{a_1}{1-r} = \frac{32}{1-\frac{1}{2}} = \boxed{64}$

(9) a. $\frac{54}{18} = \frac{162}{54} = \frac{486}{162} = 3$ (show they all have same common ratio)

b. i. $\boxed{u_n = 18(3)^{n-1}}$

ii. $1062882 = 18(3)^{n-1}$

$59049 = 3^{n-1}$

$3^{n-1} = 59049$

$\log_3 59049 = n-1$

$10 = n-1$

$\boxed{11 = n}$

(10) $r = \frac{\frac{-4}{9}}{\frac{2}{3}} = \frac{-2}{3}$, $S_\infty = \frac{\frac{2}{3}}{1-\frac{-2}{3}} = \boxed{2/5}$

(11) $u_1 = 18$, $u_3 = 8$

$8 = 18r^{3-1}$

$\frac{4}{9} = r^2$

$\pm\frac{2}{3} = r$

For $r = \frac{2}{3}$,

$S_\infty = \frac{18}{1-\frac{2}{3}} = \boxed{54}$

For $r = \frac{-2}{3}$,

$S_\infty = \frac{18}{1-\frac{-2}{3}} = \boxed{\frac{54}{5}}$

(12) a. $r = \frac{270}{405} = \boxed{\frac{2}{3}}$

b. $u_{15} = 405\left(\frac{2}{3}\right)^{14} = 1.39$ (3 sig figs)

c. $S_\infty = \frac{405}{1-\frac{2}{3}} = \boxed{1215}$

(13) a. $\sum_{r=4}^{7} 2^r = \boxed{2^4 + 2^5 + 2^6 + 2^7}$

b. $2^4 + 2^5 + 2^6 + \cdots + 2^{30}$ is a geometric series.

$r = 2$.

$S_{27} = \frac{2^4(1-2^{27})}{(1-2)} = \boxed{2147483632}$