3 **a** $u_1 = 3$ **b** $r = \frac{1}{3}$ **c** $u_5 = \frac{1}{27}$

4 **a** 3069 **b** $\frac{4095}{1024} \approx 4.00$ **c** $-134\,217\,732$

5 **c** \$26 361.59

6 **a** $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}$ **b** $S_n = \dfrac{2^n - 1}{2^n}$

c $1 - (\frac{1}{2})^n = \dfrac{2^n - 1}{2^n}$ **d** as $n \to \infty$, $S_n \to 1$

e As $n \to \infty$, the sum of the fractions approaches the area of a 1×1 unit square.

EXERCISE 6G.2

1 **a** **i** $u_1 = \frac{3}{10}$ **ii** $r = 0.1$ **b** $S = \frac{1}{3}$

2 **a** $\frac{4}{9}$ **b** $\frac{16}{99}$ **c** $\frac{104}{333}$ **4** **a** 54 **b** 14.175

5 **a** 1 **b** $4\frac{2}{7}$ **6** $u_1 = 9$, $r = \frac{2}{3}$

7 $u_1 = 8$, $r = \frac{1}{5}$ and $u_1 = 2$, $r = \frac{4}{5}$

8 **b** $S_n = 19 - 20 \times 0.9^n$ **c** 19 seconds

REVIEW SET 6A

1 **a** arithmetic, $d = -8$

b geometric, $r = 1$ *or* arithmetic, $d = 0$

c geometric, $r = -\frac{1}{2}$ **d** neither **e** arithmetic, $d = 4$

2 $k = -\frac{11}{2}$ **3** $u_n = 33 - 5n$, $S_n = \frac{n}{2}(61 - 5n)$

4 $k = \pm\frac{2\sqrt{3}}{3}$ **5** $u_n = \frac{1}{6} \times 2^{n-1}$ or $-\frac{1}{6} \times (-2)^{n-1}$

6 21, 19, 17, 15, 13, 11

7 **a** $u_n = 89 - 3n$ **b** $u_n = \dfrac{2n+1}{n+3}$

c $u_n = 100(0.9)^{n-1}$

8 **a** $1 + 4 + 9 + 16 + 25 + 36 + 49 = 140$

b $\frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} + \frac{8}{7} + \frac{9}{8} + \frac{10}{9} + \frac{11}{10} \approx 9.43$

9 **a** $10\frac{4}{5}$ **b** $16 + 8\sqrt{2}$ **10** 27 metres

11 **a** $u_n = 3n + 1$

REVIEW SET 6B

1 **b** $u_1 = 6$, $r = \frac{1}{2}$ **c** 0.000 183

2 **a** 81 **b** $-1\frac{1}{2}$ **c** 375

3 **a** 1587 **b** $47\frac{253}{256} \approx 48.0$ **4** $u_{12} = 10\,240$

5 **a** €8415.31 **b** €8488.67 **c** €8505.75

6 **a** 42 **b** $u_{n+1} - u_n = 5$ **c** $d = 5$ **d** 1672

7 $u_n = \left(\frac{3}{4}\right) 2^{n-1}$ **a** 49 152 **b** 24 575.25

8 $u_{11} = \frac{8}{19\,683} \approx 0.000\,406$ **9** **a** 17 **b** $255\frac{511}{512} \approx 256$

10 **a** $\frac{1331}{2100} \approx 0.634$ **b** $6\frac{8}{15}$ **11** \$13 972.28

12 **a** ≈ 3470 **b** Year 2029

REVIEW SET 6C

1 **a** $d = -5$ **b** $u_1 = 63$, $d = -5$ **c** -117

d $u_{54} = -202$

2 **a** $u_1 = 3$, $r = 4$ **b** $u_n = 3 \times 4^{n-1}$, $u_9 = 196\,608$

3 $u_n = 73 - 6n$, $u_{34} = -131$

4 **a** $\displaystyle\sum_{k=1}^{n} (7k - 3)$ **b** $\displaystyle\sum_{k=1}^{n} \left(\frac{1}{2}\right)^{k+1}$

5 **a** 70 **b** ≈ 241 **c** $\frac{64}{1875}$

6 12 **7** **a** £18 726.65 **b** £18 855.74

8 **a** $u_1 = 54$, $r = \frac{2}{3}$ and $u_1 = 150$, $r = -\frac{2}{5}$

b $|r| < 1$ in both cases, so the series will converge.

For $u_1 = 54$, $r = \frac{2}{3}$, $S = 162$

For $u_1 = 150$, $r = -\frac{2}{5}$, $S = 107\frac{1}{7}$

9 **a** 35.5 km **b** 1183 km **10** **a** $0 < x < 1$ **b** $35\frac{5}{7}$

EXERCISE 7A

1 **a** $p^3 + 3p^2q + 3pq^2 + q^3$ **b** $x^3 + 3x^2 + 3x + 1$

c $x^3 - 9x^2 + 27x - 27$ **d** $8 + 12x + 6x^2 + x^3$

e $27x^3 - 27x^2 + 9x - 1$ **f** $8x^3 + 60x^2 + 150x + 125$

g $8a^3 - 12a^2b + 6ab^2 - b^3$ **h** $27x^3 - 9x^2 + x - \frac{1}{27}$

i $8x^3 + 12x + \dfrac{6}{x} + \dfrac{1}{x^3}$

2 **a** $1 + 4x + 6x^2 + 4x^3 + x^4$

b $p^4 - 4p^3q + 6p^2q^2 - 4pq^3 + q^4$

c $x^4 - 8x^3 + 24x^2 - 32x + 16$

d $81 - 108x + 54x^2 - 12x^3 + x^4$

e $1 + 8x + 24x^2 + 32x^3 + 16x^4$

f $16x^4 - 96x^3 + 216x^2 - 216x + 81$

g $16x^4 + 32x^3b + 24x^2b^2 + 8xb^3 + b^4$

h $x^4 + 4x^2 + 6 + \dfrac{4}{x^2} + \dfrac{1}{x^4}$

i $16x^4 - 32x^2 + 24 - \dfrac{8}{x^2} + \dfrac{1}{x^4}$

3 **a** $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

b $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

c $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

d $x^5 - 5x^3 + 10x - \dfrac{10}{x} + \dfrac{5}{x^3} - \dfrac{1}{x^5}$

4 **a** 1 6 15 20 15 6 1

b **i** $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

ii $64x^6 - 192x^5 + 240x^4 - 160x^3 + 60x^2 - 12x + 1$

iii $x^6 + 6x^4 + 15x^2 + 20 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6}$

5 **a** $a = 2$ and $b = e^x$ **b** $T_3 = 6e^{2x}$ and $T_4 = e^{3x}$

6 **a** $7 + 5\sqrt{2}$ **b** $161 + 72\sqrt{5}$ **c** $232 - 164\sqrt{2}$

7 **a** $64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$

b 65.944 160 601 201

8 $2x^5 + 11x^4 + 24x^3 + 26x^2 + 14x + 3$ **9** **a** 270 **b** 4320

EXERCISE 7B

1 $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, $7! = 5040$, $8! = 40\,320$, $9! = 362\,880$, $10! = 3\,628\,800$

2 **a** 30 **b** 100 **c** 21 **3** **a** n **b** $(n+2)(n+1)$

4 **a** 3 **b** 6 **c** 35 **d** 210

5 **a** **i** 28 **ii** 28

b The 3rd and 7th elements in row 8 of Pascal's triangle are the same due to its symmetry.

EXERCISE 7C

1 **a** $1^{11} + \binom{11}{1}(2x)^1 + \binom{11}{2}(2x)^2 + \ldots + \binom{11}{10}(2x)^{10} + (2x)^{11}$

b $(3x)^{15} + \binom{15}{1}(3x)^{14}\left(\frac{2}{x}\right)^1 + \binom{15}{2}(3x)^{13}\left(\frac{2}{x}\right)^2 + \ldots$

$\ldots + \binom{15}{14}(3x)^1\left(\frac{2}{x}\right)^{14} + \left(\frac{2}{x}\right)^{15}$

c $(2x)^{20} + \binom{20}{1}(2x)^{19}\left(-\frac{3}{x}\right)^1 + \binom{20}{2}(2x)^{18}\left(-\frac{3}{x}\right)^2 + \ldots$

$\ldots + \binom{20}{19}(2x)^1\left(-\frac{3}{x}\right)^{19} + \left(-\frac{3}{x}\right)^{20}$

2 **a** $T_6 = \binom{15}{5}(2x)^{10}5^5$ **b** $T_4 = \binom{9}{3}(x^2)^6y^3$

c $T_{10} = \binom{17}{9}x^8\left(-\frac{2}{x}\right)^9$ **d** $T_9 = \binom{21}{8}(2x^2)^{13}\left(-\frac{1}{x}\right)^8$