REVIEW SET 6A NON-CALCULATOR

1 Identify the following sequences as arithmetic, geometric, or neither:

a $7, -1, -9, -17, ...$ **b** $9, 9, 9, 9, ...$ **c** $4, -2, 1, -\frac{1}{2}, ...$

d $1, 1, 2, 3, 5, 8, ...$ **e** the set of all multiples of 4 in ascending order.

2 Find k if $3k$, $k-2$, and $k+7$ are consecutive terms of an arithmetic sequence.

3 Show that $28, 23, 18, 13, ...$ is an arithmetic sequence. Hence find u_n and the sum S_n of the first n terms in simplest form.

4 Find k given that 4, k, and k^2-1 are consecutive terms of a geometric sequence.

5 Determine the general term of a geometric sequence given that its sixth term is $\frac{16}{3}$ and its tenth term is $\frac{256}{3}$.

6 Insert six numbers between 23 and 9 so that all eight numbers are in arithmetic sequence.

7 Find, in simplest form, a formula for the general term u_n of:

a $86, 83, 80, 77, ...$ **b** $\frac{3}{4}, 1, \frac{7}{6}, \frac{9}{7}, ...$ **c** $100, 90, 81, 72.9, ...$

Hint: One of these sequences is neither arithmetic nor geometric.

8 Expand and hence evaluate: **a** $\sum_{k=1}^{7} k^2$ **b** $\sum_{k=1}^{8} \frac{k+3}{k+2}$

9 Find the sum of each of the following infinite geometric series:

a $18 - 12 + 8 - ....$ **b** $8 + 4\sqrt{2} + 4 + ...$

10 A ball bounces from a height of 3 metres and returns to 80% of its previous height on each bounce. Find the total distance travelled by the ball until it stops bouncing.

11 The sum of the first n terms of an infinite sequence is $\frac{3n^2+5n}{2}$ for all $n \in \mathbb{Z}^+$.

a Find the nth term. **b** Prove that the sequence is arithmetic.

REVIEW SET 6B CALCULATOR

1 A sequence is defined by $u_n = 6(\frac{1}{2})^{n-1}$.

a Prove that the sequence is geometric. **b** Find u_1 and r.

c Find the 16th term of the sequence to 3 significant figures.

2 Consider the sequence $24, 23\frac{1}{4}, 22\frac{1}{2}, ...$

a Which term of the sequence is -36? **b** Find the value of u_{35}.

c Find S_{40}, the sum of the first 40 terms of the sequence.

3 Find the sum of:

a the first 23 terms of $3 + 9 + 15 + 21 + ...$

b the first 12 terms of $24 + 12 + 6 + 3 + ...$

4 Find the first term of the sequence $5, 10, 20, 40, ...$ which exceeds 10 000.

5 What will an investment of €6000 at 7% p.a. compound interest amount to after 5 years if the interest is compounded:

a annually **b** quarterly **c** monthly?

6 The nth term of a sequence is given by the formula $u_n = 5n - 8$.

a Find the value of u_{10}.

b Write down an expression for $u_{n+1} - u_n$ and simplify it.

c Hence explain why the sequence is arithmetic.

d Evaluate $u_{15} + u_{16} + u_{17} + ... + u_{30}$.

7 A geometric sequence has $u_6 = 24$ and $u_{11} = 768$. Determine the general term of the sequence and hence find:

a u_{17} **b** the sum of the first 15 terms.

8 Find the first term of the sequence $24, 8, \frac{8}{3}, \frac{8}{9}, ...$ which is less than 0.001.

9 **a** Determine the number of terms in the sequence $128, 64, 32, 16, ..., \frac{1}{512}$.

b Find the sum of these terms.

10 Find the sum of each of the following infinite geometric series:

a $1.21 - 1.1 + 1 - ....$ **b** $\frac{14}{3} + \frac{4}{3} + \frac{8}{21} + ...$

11 How much should be invested at a fixed rate of 9% p.a. compound interest if you need it to amount to \$20 000 after 4 years with interest paid monthly?

12 In 2004 there were 3000 iguanas on a Galapagos island. Since then, the population of iguanas on the island has increased by 5% each year.

a How many iguanas were on the island in 2007?

b In what year will the population first exceed 10 000?

REVIEW SET 6C

1 A sequence is defined by $u_n = 68 - 5n$.

a Prove that the sequence is arithmetic. **b** Find u_1 and d.

c Find the 37th term of the sequence.

d State the first term of the sequence which is less than -200.

2 **a** Show that the sequence $3, 12, 48, 192, ...$ is geometric.

b Find u_n and hence find u_9.

3 Find the general term of the arithmetic sequence with $u_7 = 31$ and $u_{15} = -17$. Hence, find the value of u_{34}.

4 Write using sigma notation:

a $4 + 11 + 18 + 25 + ...$ for n terms **b** $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$ for n terms.

5 Evaluate:

a $\sum_{k=1}^{8} \left(\frac{31 - 3k}{2}\right)$ **b** $\sum_{k=1}^{15} 50(0.8)^{k-1}$ **c** $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1}$

6 How many terms of the series $11 + 16 + 21 + 26 + ...$ are needed to exceed a sum of 450?

7 £12 500 is invested in an account which pays 8.25% p.a. compounded. Find the value of the investment after 5 years if the interest is compounded:

a half-yearly **b** monthly.

8 The sum of the first two terms of an infinite geometric series is 90. The third term is 24.

a Show that there are two possible series. Find the first term and the common ratio in each case.

b Show that both series converge and find their respective sums.

9 Seve is training for a long distance walk. He walks for 10 km in the first week, then each week thereafter he walks an additional 500 m. If he continues this pattern for a year, how far does Seve walk:

a in the last week **b** in total?

10 **a** Under what conditions will the series $\sum_{k=1}^{\infty} 50(2x-1)^{k-1}$ converge?

Explain your answer.

b Find $\sum_{k=1}^{\infty} 50(2x-1)^{k-1}$ if $x = 0.3$.