Sequences & Series Review

① $u_1 = 2 \quad u_5 = 32$

$S_5 = \frac{5}{2}(2+32) = \frac{5}{2}(34) = 85$

② $1990 \Rightarrow u_1 = 160$ a) $r = \frac{u_2}{u_1} = \frac{240}{160} = \frac{3}{2}$

$1991 \Rightarrow u_2 = 240$

b) $2002 \Rightarrow u_{13}$ $u_{13} = u_1(r)^{13-1}$

$u_{13} = 160\left(\frac{3}{2}\right)^{12}$

$u_{13} = 20759$

c) $5000 = 160\left(\frac{3}{2}\right)^{n-1}$

$31.25 = \left(\frac{3}{2}\right)^{n-1}$

$\log_{3/2}(31.25) = n-1$

$8.48906 = n-1$

$9.49 = n$, which goes into the 10^{th} year, so 1999.

d) $S_{13} = \frac{160\left(\frac{3}{2}^{13} - 1\right)}{\frac{3}{2} - 1} = 61958$

e) A lot of the population would have one, so there would be fewer people to buy one.

(3) a) Ashley: $u_1 = 12$, $u_2 = 14$, $u_3 = 16$
(arithmetic, $d = 2$)
$S_{15} = \frac{15}{2}\left(2(12) + (15-1)2\right) = 390$ hours

b) Billie: $u_1 = 12$, $u_2 = 13.2$, $u_3 = 14.52$
(geometric, $r = 1.1$)
i) $u_3 = u_1(1.1)^{3-1}$
$u_3 = 12(1.1)^2 = 14.52$
ii) $S_{15} = \frac{12(1.1^{15} - 1)}{1.1 - 1} = 381$ hours

c) Graph $y_1 = 12(1.1)^{x-1}$
$y_2 = 50$
Find intersection. In 16^{th} week.

(4) $u_1 = -2$ $u_4 = 16$ $u_n = 11998$

a) $u_4 = u_1 + (4-1)d$
$16 = -2 + 3d$
$18 = 3d$
$6 = d$

b) $u_n = u_1 + (n-1)d$
$11998 = -2 + (n-1)(6)$
$12000 = 6n - 6$
$12006 = 6n$
$2001 = n$

(5) If $S_1 = 7$, then $u_1 = 7$. $S_2 = 18$, so $u_2 = 18 - 7 = 11$

a) $u_1 = 7$
b) $d = u_2 - u_1 = 11 - 7 = 4$
c) $u_4 = u_1 + (4-1)(4)$
$u_4 = 7 + 3(4) = 19$

(6) a) $A = 5000(1+.063)^n$

b) $A = 5000(1.063)^5 = \$6786.35$

c) i. $5000(1.063)^n > 10{,}000$

ii. $n = 12$ years

(7) a) $u_8 = u_1 + (8-1)d$ $\quad u_4 = u_1 + (4-1)d$

$30 = u_1 + 7d$ $\quad 12 = u_1 + 3d$

$12 - 3d = u_1$

$30 = (12-3d) + 7d$

$18 = 4d$

$\frac{9}{2} = d$ $\quad u_n = -\frac{3}{2} + (n-1)\left(\frac{9}{2}\right)$

$u_1 = -\frac{3}{2}$

b) $u_8 = u_1(r)^{8-1}$ $\quad u_4 = u_1(r)^{4-1}$

$30 = u_1(r)^7$ $\quad 12 = u_1(r)^3$

$\frac{30}{r^7} = u_1$ $\quad \frac{12}{r^3} = u_1$

$$\frac{30}{r^7} = \frac{12}{r^3}$$

$$30r^3 = 12r^7$$

$$\frac{5}{2} = r^4$$

$r = \pm\sqrt[4]{\frac{5}{2}}$, so $u_1 = \pm 6.04$

$r = \pm 1.26$ (3 sig figs)

$$u_n = 6.04(1.26)^{n-1}$$

or

$$u_n = -6.04(-1.26)^{n-1}$$

⑧ a) $u_{21} = u_1 + (21-1)d$ $\qquad$ $u_4 = u_1 + (4-1)d$

$-37 = u_1 + 20d$ $\qquad$ $-3 = u_1 + 3d$

$-37 \mp 20d = u_1$ $\qquad$ $-3 + -3d = u_1$

$$-37 \mp 20d = -3 - 3d$$
$$-17d = 34$$
$$d = -2$$
$$u_1 = 3$$

b) $S_{10} = \frac{10}{2}\left(2(3) + (10-1)(-2)\right) = -60$

⑨ This is an arithmetic pattern, since we add 3 to get to the next term. We need to know which term 3750 is.

$$u_n = 3 + (n-1)(3)$$
$$3750 = 3 + 3n - 3$$
$$1250 = n$$

$$S_{1250} = \frac{1250}{2}(3 + 3750) = 2{,}345{,}625$$

⑩ $u_1 = 18$ $\quad$ $u_3 = 8$ (geometric)

$$u_3 = u_1 (r)^{3-1}$$
$$8 = 18r^2$$
$$\frac{4}{9} = r^2$$
$$\pm\frac{2}{3} = r$$

If $r = \frac{2}{3}$, $S_\infty = \dfrac{18}{1-\frac{2}{3}} = 54$

If $r = -\frac{2}{3}$, $S_\infty = \dfrac{18}{1-\frac{-2}{3}} = 10.8$