**7**

The graphs of $y = x^2 - 3x$ and $y = 2x - x^2$ are illustrated.

a Without using technology, show that the graphs meet where $x = 0$ and $x = 2\frac{1}{2}$.

b Find the maximum vertical separation between the curves for $0 \leqslant x \leqslant 2\frac{1}{2}$.

8 Infinitely many rectangles may be inscribed within the right angled triangle shown alongside. One of them is illustrated.

a Let $AB = x$ cm and $BC = y$ cm. Use similar triangles to find y in terms of x.

b Find the dimensions of rectangle ABCD of maximum area.
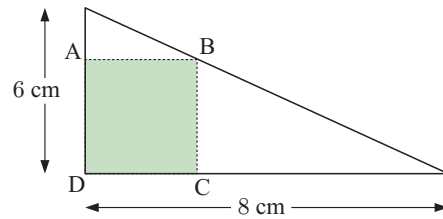
## INVESTIGATION 4 — SUM AND PRODUCT OF ROOTS

What to do:

1 Suppose $ax^2 + bx + c = 0$ has roots p and q.

Prove that $p + q = \dfrac{-b}{a}$ and $pq = \dfrac{c}{a}$.

2 Suppose $2x^2 - 5x + 1 = 0$ has roots p and q. Without finding the values of p and q, find:

a $p + q$ **b** pq **c** $p^2 + q^2$ **d** $\dfrac{1}{p} + \dfrac{1}{q}$

3 Find *all* quadratic equations with roots which are:

a one more than the roots of $2x^2 - 5x + 1 = 0$

b the squares of the roots of $2x^2 - 5x + 1 = 0$

c the reciprocals of the roots of $2x^2 - 5x + 1 = 0$.

REVIEW SET 1A — NON-CALCULATOR

1 Consider the quadratic function $y = -2(x + 2)(x - 1)$.

a State the x-intercepts. **b** State the equation of the axis of symmetry.

c Find the y-intercept. **d** Find the coordinates of the vertex.

e Sketch the function.

2 Solve the following equations, giving exact answers:

a $3x^2 - 12x = 0$ **b** $3x^2 - x - 10 = 0$ **c** $x^2 - 11x = 60$

3 Solve using the quadratic formula:

a $x^2 + 5x + 3 = 0$ **b** $3x^2 + 11x - 2 = 0$

4 Solve by 'completing the square': $x^2 + 7x - 4 = 0$

5 Use the vertex, axis of symmetry, and y-intercept to graph:

     a $y = (x - 2)^2 - 4$ **b** $y = -\frac{1}{2}(x + 4)^2 + 6$

6 Find, in the form $y = ax^2 + bx + c$,   the equation of the quadratic whose graph:

     a touches the x-axis at 4 and passes through $(2, 12)$

     b has vertex $(-4, 1)$ and passes through $(1, 11)$.

7 Find the maximum or minimum value of the relation $y = -2x^2 + 4x + 3$ and the value of x at which this occurs.

8 Find the points of intersection of $y = x^2 - 3x$ and $y = 3x^2 - 5x - 24$.

9 For what values of k does the graph of $y = -2x^2 + 5x + k$ not cut the x-axis?

10 Find the values of m for which $2x^2 - 3x + m = 0$ has:

     a a repeated root   **b** two distinct real roots   **c** no real roots.

11 The sum of a number and its reciprocal is $2\frac{1}{30}$. Find the number.

12 Show that no line with a y-intercept of $(0, 10)$ will ever be tangential to the curve with equation $y = 3x^2 + 7x - 2$.

**13** The diagram shows a quadratic $y = x^2 + mx + n$.
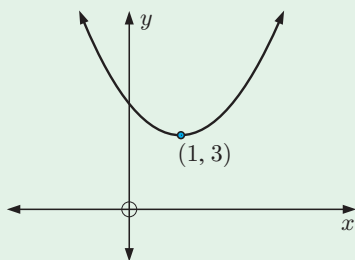
     a Determine the values of m and n.

     b Find k given that the graph passes through the point $(3, k)$.

$(1, 3)$

REVIEW SET 1B   CALCULATOR

1 Consider the quadratic function $y = 2x^2 + 6x - 3$.

     a Convert it to the form $y = a(x - h)^2 + k$.

     b State the coordinates of the vertex.

     c Find the y-intercept.

     d Sketch the graph of the function.

2 Solve:

     a $(x - 2)(x + 1) = 3x - 4$ **b** $2x - \dfrac{1}{x} = 5$

3 Draw the graph of $y = -x^2 + 2x$.

4 Consider the quadratic function $y = -3x^2 + 8x + 7$.   Find the equation of the axis of symmetry, and the coordinates of the vertex.
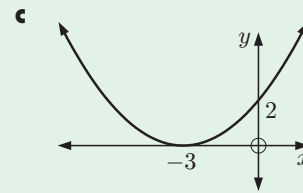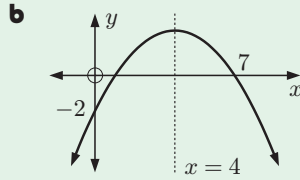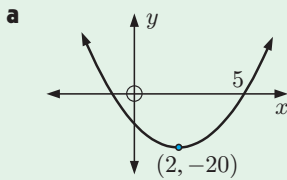
5 Using the discriminant only, determine the nature of the solutions of:

    a $2x^2 - 5x - 7 = 0$ **b** $3x^2 - 24x + 48 = 0$

6   **a** For what values of c do the lines with equations $y = 3x + c$ intersect the parabola $y = x^2 + x - 5$ in two distinct points?

    b Choose one such value of c from part **a** and find the points of intersection in this case.

7 Consider the quadratic function $y = 2x^2 + 4x - 1$.

    a State the axis of symmetry.   **b** Find the coordinates of the vertex.

    c Find the axes intercepts.   **d** Hence sketch the function.

8 An open square-based container is made by cutting 4 cm square pieces out of a piece of tinplate. If the volume of the container is 120 cm^3, find the size of the original piece of tinplate.

9 Consider $y = -x^2 - 5x + 3$ and $y = x^2 + 3x + 11$.

    a Solve for x: $-x^2 - 5x + 3 = x^2 + 3x + 11$.

    b Hence, or otherwise, determine the values of x for which $x^2 + 3x + 11 > -x^2 - 5x + 3$.

10 Find the maximum or minimum value of the following quadratics, and the corresponding value of x:

    a $y = 3x^2 + 4x + 7$ **b** $y = -2x^2 - 5x + 2$

11 600 m of fencing is used to construct 6 rectangular animal pens as shown.

    a Show that the area A of each pen is
$$A = x\left(\frac{600 - 8x}{9}\right) \text{ m}^2.$$

    b Find the dimensions of each pen so that it has the maximum possible area.

    c What is the area of each pen in this case?

12 Two different quadratic functions of the form $y = 9x^2 - kx + 4$ each touch the x-axis.

    a Find the two values of k.

    b Find the point of intersection of the two quadratic functions.
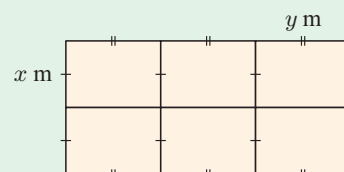
REVIEW SET 1C

1 Consider the quadratic function $y = \frac{1}{2}(x - 2)^2 - 4$.

    a State the equation of the axis of symmetry.

    b Find the coordinates of the vertex.   **c** Find the y-intercept.

    d Sketch the function.

2 Solve the following equations:

    a $x^2 - 5x - 3 = 0$ **b** $2x^2 - 7x - 3 = 0$

3 Solve the following using the quadratic formula:

    a $x^2 - 7x + 3 = 0$ **b** $2x^2 - 5x + 4 = 0$

4 Find the equation of the quadratic function with graph:

a

(2, −20)

b

−2   7

$x = 4$

c

2

−3

5 Use the discriminant only to find the relationship between the graph and the x-axis for:

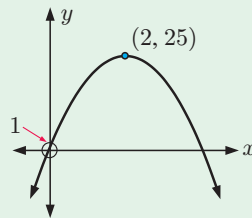**a** $y = 2x^2 + 3x - 7$

b $y = -3x^2 - 7x + 4$

6 Determine whether the following quadratic functions are positive definite, negative definite, or neither:

a $y = -2x^2 + 3x + 2$

b $y = 3x^2 + x + 11$

7 Find the equation of the quadratic function shown:


(2, 25)

1

8 Find the y-intercept of the line with gradient -3 that is tangential to the parabola $y = 2x^2 - 5x + 1$.

9 For what values of k would the graph of $y = x^2 - 2x + k$ cut the x-axis twice?

10 Find the quadratic function which cuts the x-axis at 3 and -2 and which has y-intercept 24. Give your answer in the form $y = ax^2 + bx + c$.

11 For what values of m are the lines $y = mx - 10$ tangents to the parabola $y = 3x^2 + 7x + 2$?

12 The diagram shows the parabola $y = a(x + m)(x + n)$ where $m > n$.

A   B
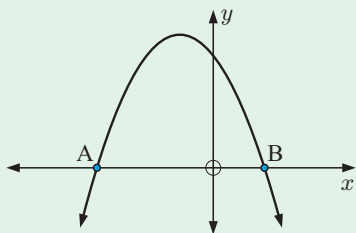
a Find, in terms of m and n, the:

  i coordinates of the x-intercepts A and B

  ii equation of the axis of symmetry.

b State the sign of:

  i the discriminant Δ

  ii a.

13 **a** Determine the equation of:

  i the quadratic function

  ii the straight line.

  b For what values of x is the straight line above the curve?

−3   3

−27