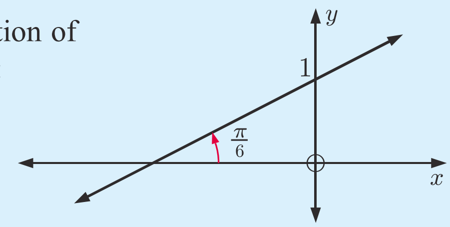
Example 17 — Self Tutor

Find the equation of the given line:



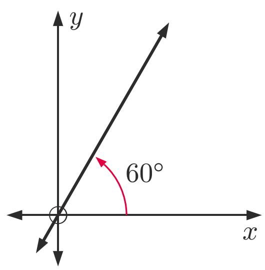
The line has gradient $m = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ and y-intercept 1.

$\therefore$ the line has equation $y = \frac{1}{\sqrt{3}}x + 1$.

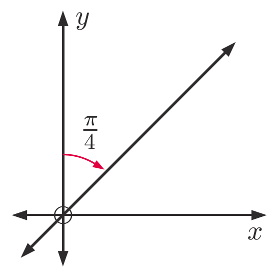
EXERCISE 8F

1 Find the equation of each line:

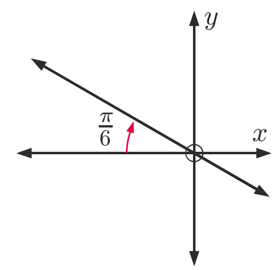
a



b

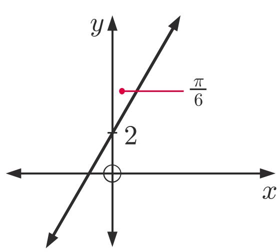


c

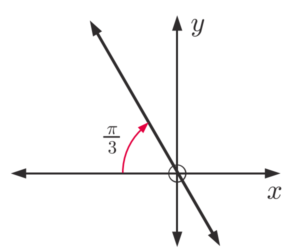


2 Find the equation of each line:

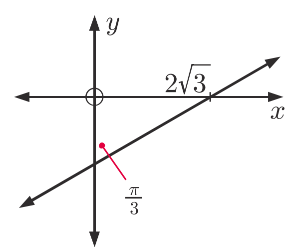
a



b



c



REVIEW SET 8A — NON-CALCULATOR

1 Convert these to radians in terms of π:

a $120°$ **b** $225°$ **c** $150°$ **d** $540°$

2 Find the acute angles that would have the same:

a sine as $\frac{2\pi}{3}$ **b** sine as $165°$ **c** cosine as $276°$.

3 Find:

a $\sin 159°$ if $\sin 21° \approx 0.358$ **b** $\cos 92°$ if $\cos 88° \approx 0.035$

c $\cos 75°$ if $\cos 105° \approx -0.259$ **d** $\sin(-133°)$ if $\sin 47° \approx 0.731$.

4 Use a unit circle diagram to find:

a $\cos 360°$ and $\sin 360°$ **b** $\cos(-\pi)$ and $\sin(-\pi)$.

5 Explain how to use the unit circle to find θ when $\cos\theta = -\sin\theta$, $0 \leqslant \theta \leqslant 2\pi$.

6 Find exact values for $\sin\theta$, $\cos\theta$, and $\tan\theta$ for θ equal to:

a $\frac{2\pi}{3}$ **b** $\frac{8\pi}{3}$

7 If $\sin x = -\frac{1}{4}$ and $\pi < x < \frac{3\pi}{2}$, find $\tan x$ exactly.

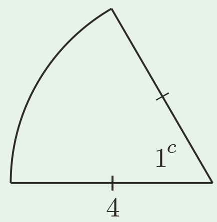
8 If $\cos\theta = \frac{3}{4}$ find the possible values of $\sin\theta$.

9 Evaluate:

a $2\sin(\frac{\pi}{3})\cos(\frac{\pi}{3})$ **b** $\tan^2(\frac{\pi}{4}) - 1$ **c** $\cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6})$

10 Given $\tan x = -\frac{3}{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find: **a** $\sin x$ **b** $\cos x$.

11



Find the perimeter and area of the sector.

12 Suppose $\cos\theta = \frac{\sqrt{11}}{\sqrt{17}}$ and θ is acute. Find the exact value of $\tan\theta$.

REVIEW SET 8B CALCULATOR

1 Determine the coordinates of the point on the unit circle corresponding to an angle of:

a $320°$ **b** $163°$

2 Convert to radians to 4 significant figures:

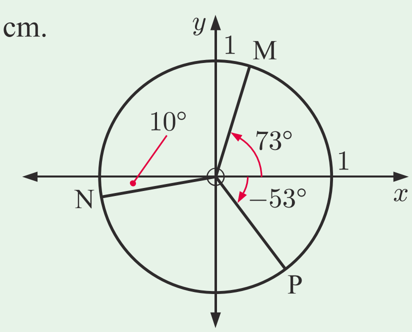
a $71°$ **b** $124.6°$ **c** $-142°$

3 Convert these radian measurements to degrees, to 2 decimal places:

a 3 **b** 1.46 **c** 0.435 **d** -5.271

4 Determine the area of a sector of angle $\frac{5\pi}{12}$ and radius 13 cm.

5 Find the coordinates of the points M, N, and P on the unit circle.



6 Find the angle [OA] makes with the positive x-axis if the x-coordinate of the point A on the unit circle is -0.222.

7 Find all angles between $0°$ and $360°$ which have:

a a cosine of $-\frac{\sqrt{3}}{2}$ **b** a sine of $\frac{1}{\sqrt{2}}$ **c** a tangent of $-\sqrt{3}$

8 Find θ for $0 \leqslant \theta \leqslant 2\pi$ if:

a $\cos\theta = -1$ **b** $\sin^2\theta = \frac{3}{4}$

9 Find the obtuse angles which have the same:

a sine as $47°$ **b** sine as $\frac{\pi}{15}$ **c** cosine as $186°$

10 Find the perimeter and area of a sector of radius 11 cm and angle $63°$.

11 Find the radius and area of a sector of perimeter 36 cm with an angle of $\frac{2\pi}{3}$.

12 Find two angles on the unit circle with $0 \leqslant \theta \leqslant 2\pi$, such that:

a $\cos\theta = \frac{2}{3}$ **b** $\sin\theta = -\frac{1}{4}$ **c** $\tan\theta = 3$

REVIEW SET 8C

1 Convert these radian measurements to degrees:

a $\frac{2\pi}{5}$ **b** $\frac{5\pi}{4}$ **c** $\frac{7\pi}{9}$ **d** $\frac{11\pi}{6}$

2 Illustrate the regions where $\sin\theta$ and $\cos\theta$ have the same sign.

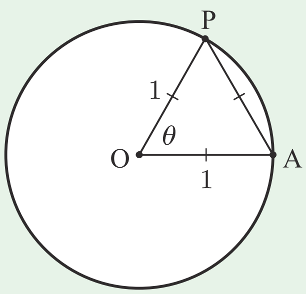
3 Use a unit circle diagram to find:

a $\cos(\frac{3\pi}{2})$ and $\sin(\frac{3\pi}{2})$ **b** $\cos(-\frac{\pi}{2})$ and $\sin(-\frac{\pi}{2})$

4 Suppose $m = \sin p$, where p is acute. Write an expression in terms of m for:

a $\sin(\pi - p)$ **b** $\sin(p + 2\pi)$ **c** $\cos p$ **d** $\tan p$

5



a State the value of θ in:

i degrees **ii** radians.

b State the arc length AP.

c State the area of the minor *sector* OAP.

6 Without a calculator, evaluate $\tan^2(\frac{2\pi}{3})$.

7 Show that $\cos(\frac{3\pi}{4}) - \sin(\frac{3\pi}{4}) = -\sqrt{2}$.

8 If $\cos\theta = -\frac{3}{4}$, $\frac{\pi}{2} < \theta < \pi$ find the exact value of:

a $\sin\theta$ **b** $\tan\theta$ **c** $\sin(\theta + \pi)$

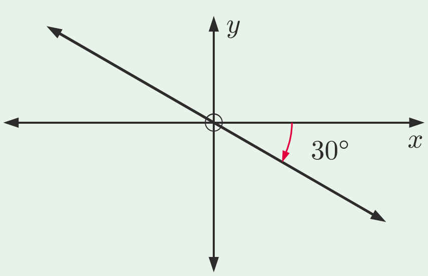
9 Without using a calculator, evaluate:

a $\tan^2 60° - \sin^2 45°$ **b** $\cos^2(\frac{\pi}{4}) + \sin(\frac{\pi}{2})$ **c** $\cos(\frac{5\pi}{3}) - \tan(\frac{5\pi}{4})$

10 Simplify:

a $\sin(\pi - \theta) - \sin\theta$ **b** $\cos\theta\tan\theta$

11



a Find the equation of the line drawn.

b Find the exact value of k given the point $(k, 2)$ lies on the line.

12 Three circles with radius r are drawn as shown, each with its centre on the circumference of the other two circles. A, B and C are the centres of the three circles.

Prove that an expression for the area of the shaded region is $A = \frac{r^2}{2}(\pi - \sqrt{3})$.