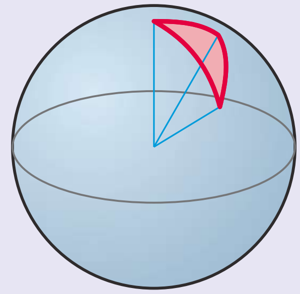
THEORY OF KNOWLEDGE

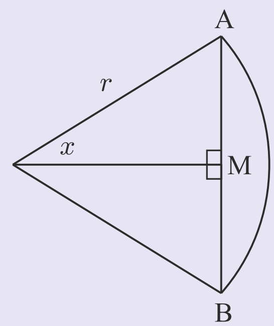
Trigonometry appears to be one of the most useful disciplines of mathematics, having great importance in building and engineering. Its study has been driven by the need to solve real world problems throughout history.



The study of trigonometry began when Greek, Babylonian, and Arabic astronomers needed to calculate the positions of stars and planets. These early mathematicians considered the trigonometry of spherical triangles, which are triangles on the surface of a sphere.

Trigonometric functions were developed by Hipparchus around 140 BC, and then by Ptolemy and Menelaus around 100 AD.

Around 500 AD, Hindu mathematicians published a table called the *Aryabhata*. It was a table of lengths of half chords, which are the lengths $AM = r\sin x$ in the diagram. This is trigonometry of triangles in a plane, as we study in schools today.



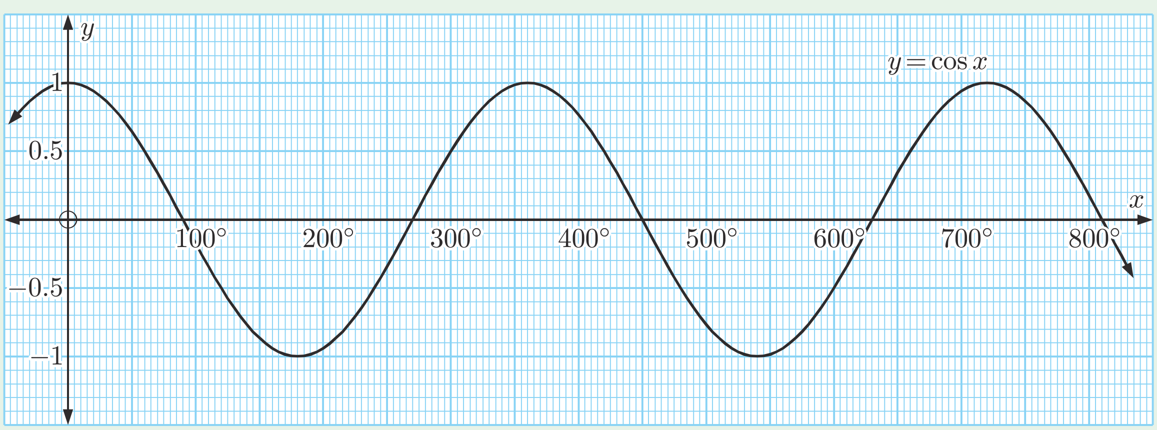
1 How do society and culture affect mathematical knowledge?

2 Should congruence and similarity, or the work of Pythagoras, be considered part of modern trigonometry?

3 Is the angle sum of a triangle always equal to $180°$?

REVIEW SET 11A — NON-CALCULATOR

1



Use the graph of $y = \cos x$ to find the solutions of:

a $\cos x = -0.4, \quad 0 \leqslant x \leqslant 800°$

b $\cos x = 0.9, \quad 0 \leqslant x \leqslant 600°$

2 Solve in terms of π:

a $2\sin x = -1$ for $0 \leqslant x \leqslant 4\pi$ **b** $\sqrt{2}\sin x - 1 = 0$ for $-2\pi \leqslant x \leqslant 2\pi$

3 Find the x-intercepts of:

a $y = 2\sin 3x + \sqrt{3}$ for $0 \leqslant x \leqslant 2\pi$ **b** $y = \sqrt{2}\sin(x + \frac{\pi}{4})$ for $0 \leqslant x \leqslant 3\pi$

4 Solve $\sqrt{2}\cos(x + \frac{\pi}{4}) - 1 = 0$ for $0 \leqslant x \leqslant 4\pi$.

5 Simplify:

a $\dfrac{1 - \cos^2\theta}{1 + \cos\theta}$ **b** $\dfrac{\sin\alpha - \cos\alpha}{\sin^2\alpha - \cos^2\alpha}$ **c** $\dfrac{4\sin^2\alpha - 4}{8\cos\alpha}$

6 If $\sin\alpha = -\frac{3}{4}$, $\pi \leqslant \alpha \leqslant \frac{3\pi}{2}$, find the value of $\cos\alpha$ and hence the value of $\sin 2\alpha$.

7 Show that $\dfrac{\sin 2\alpha - \sin\alpha}{\cos 2\alpha - \cos\alpha + 1}$ simplifies to $\tan\alpha$.

REVIEW SET 11B CALCULATOR

1 Solve for $0 \leqslant x \leqslant 8$:

a $\sin x = 0.382$ **b** $\tan(\frac{x}{2}) = -0.458$

2 Solve:

a $\cos x = 0.4379$ for $0 \leqslant x \leqslant 10$ **b** $\cos(x - 2.4) = -0.6014$ for $0 \leqslant x \leqslant 6$

3 If $\sin A = \frac{5}{13}$ and $\cos A = \frac{12}{13}$, find: **a** $\sin 2A$ **b** $\cos 2A$ **c** $\tan 2A$

4 **a** Solve for $0 \leqslant x \leqslant 10$:

i $\tan x = 4$ **ii** $\tan(\frac{x}{4}) = 4$ **iii** $\tan(x - 1.5) = 4$

b Find exact solutions for x given $-\pi \leqslant x \leqslant \pi$:

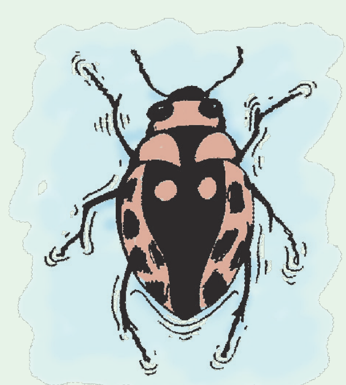
i $\tan(x + \frac{\pi}{6}) = -\sqrt{3}$ **ii** $\tan 2x = -\sqrt{3}$ **iii** $\tan^2 x - 3 = 0$

c Solve $3\tan(x - 1.2) = -2$ for $0 \leqslant x \leqslant 10$.

5 Solve for $0 \leqslant x \leqslant 2\pi$:

a $\cos x = 0.3$ **b** $2\sin(3x) = \sqrt{2}$ **c** $43 + 8\sin x = 50.1$

6 An ecologist studying a species of water beetle estimates the population of a colony over an eight week period. If t is the number of weeks after the initial estimate is made, then the population in thousands can be modelled by $P(t) = 5 + 2\sin(\frac{\pi t}{3})$ where $0 \leqslant t \leqslant 8$.



a What was the initial population?

b What were the smallest and largest populations?

c During what time interval(s) did the population exceed 6000?

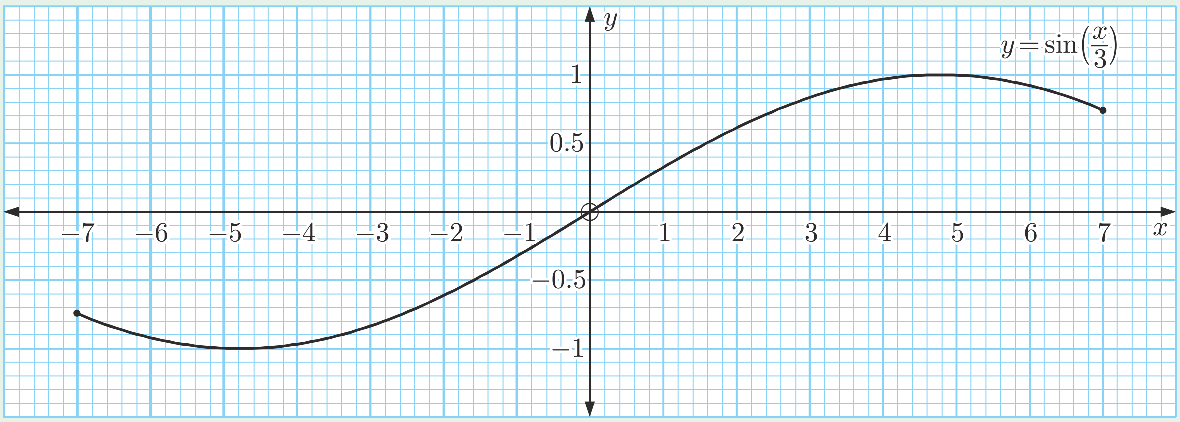
7 Solve for x: $3\cos x + \sin 2x = 1$ for $0 \leqslant x \leqslant 10$.

REVIEW SET 11C

1 Consider $y = \sin(\frac{x}{3})$ on the domain $-7 \leqslant x \leqslant 7$. Use the graph to solve, correct to 1 decimal place:

a $\sin(\frac{x}{3}) = -0.9$

b $\sin(\frac{x}{3}) = \frac{1}{4}$



2 Solve algebraically for $0 \leqslant x \leqslant 2\pi$, giving answers in terms of π:

a $\sin^2 x - \sin x - 2 = 0$

b $4\sin^2 x = 1$

3 Find the exact solutions of:

a $\tan(x - \frac{\pi}{3}) = \frac{1}{\sqrt{3}}$, $0 \leqslant x \leqslant 4\pi$

b $\cos(x + \frac{2\pi}{3}) = \frac{1}{2}$, $-2\pi \leqslant x \leqslant 2\pi$

4 Simplify:

a $\cos^3\theta + \sin^2\theta\cos\theta$

b $\dfrac{\cos^2\theta - 1}{\sin\theta}$

c $5 - 5\sin^2\theta$

d $\dfrac{\sin^2\theta - 1}{\cos\theta}$

5 Expand and simplify if possible:

a $(2\sin\alpha - 1)^2$

b $(\cos\alpha - \sin\alpha)^2$

6 Show that:

a $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{2}{\cos\theta}$

b $\left(1 + \dfrac{1}{\cos\theta}\right)\left(\cos\theta - \cos^2\theta\right) = \sin^2\theta$

7 If $\tan\theta = -\frac{2}{3}$, $\frac{\pi}{2} < \theta < \pi$, find $\sin\theta$ and $\cos\theta$ exactly.