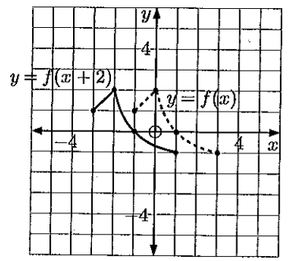
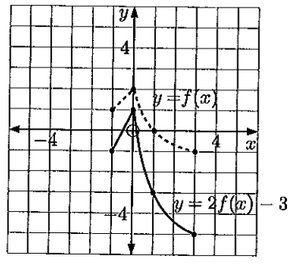
d The function which is a vertical stretch of f with scale factor 2, is

$$\begin{aligned} y &= 2f(x) \\ &= 2\left(\frac{1}{x-1} + \sqrt{x+1}\right) \\ &= \frac{2}{x-1} + 2\sqrt{x+1} \end{aligned}$$

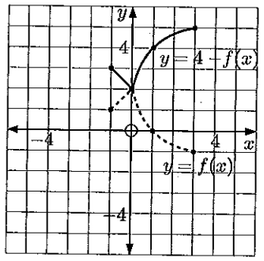
9 **a**



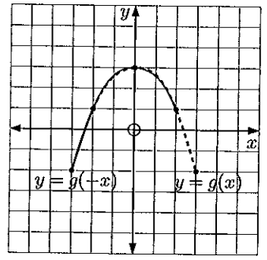
b



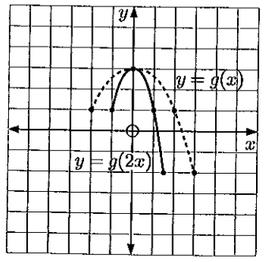
c



10 **a**



b



11 **a** Domain $= \{x \mid x > 2\}$, Range $= \{y \mid y \in \mathbb{R}\}$

b $y = g(x)$ has the vertical asymptote $x = 2$.

c $y = g(2x)$ is a horizontal stretch of g with scale factor $\frac{1}{2}$.

$\therefore\ h(x) = g(2x)$

$= 4 - \ln(2x - 2)$

or $h : x \mapsto 4 - \ln(2x - 2)$

d $y = h(x)$ is defined when $2x - 2 > 0$

$\therefore\ x > 1$

$\therefore\ y = h(x)$ has the vertical asymptote $x = 1$.

12 **a** $y = -4x(x + 3)$

i The graph cuts the x-axis when $y = 0$

$\therefore\ -4x(x + 3) = 0$

$\therefore\ x = 0$ or -3

$\therefore$ the x-intercepts are 0 and -3.

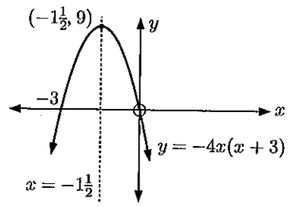
ii The axis of symmetry is $x = -1\frac{1}{2}$

iii $f(-\frac{3}{2}) = -4(-\frac{3}{2})(\frac{3}{2}) = 9$

$\therefore$ the vertex is at $(-1\frac{1}{2}, 9)$.

iv When $x = 0$, $y = 0$, so the y-intercept is 0.

v



b $y = \frac{1}{2}(x + 6)(x - 4)$

i When $y = 0$, $x = -6$ or 4

$\therefore$ the x-intercepts are -6 and 4.

ii The axis of symmetry is $x = -1$.

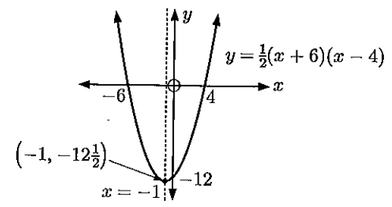
iii $f(-1) = \frac{1}{2}(5)(-5) = -12\frac{1}{2}$

$\therefore$ the vertex is at $(-1, -12\frac{1}{2})$.

iv When $x = 0$, $y = \frac{1}{2}(6)(-4) = -12$

$\therefore$ the y-intercept is -12.

v



c $y = -3(x - 2)^2$

i The graph cuts x-axis when $y = 0$

$\therefore\ (x - 2)^2 = 0$

$\therefore$ the graph touches the x-axis when $x = 2$.

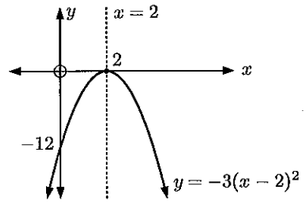
ii The axis of symmetry is $x = 2$.

iii $f(2) = 0$

$\therefore$ the vertex is at $(2, 0)$.

iv The y-intercept is $f(0) = -3(-2)^2 = -12$.

v



d $y = 2(x + 5)^2 - 4$

i When $y = 0$, $2(x + 5)^2 - 4 = 0$

$\therefore\ (x + 5)^2 = 2$

$\therefore\ x + 5 = \pm\sqrt{2}$

$\therefore\ x = -5 \pm \sqrt{2}$

$\therefore$ the x-intercepts are $-5 + \sqrt{2}$ and $-5 - \sqrt{2}$.

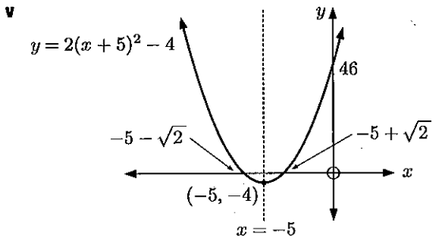
ii The axis of symmetry is $x = -5$.

iii $f(-5) = 2(0)^2 - 4 = -4$

$\therefore$ the vertex is at $(-5, -4)$.

iv The y-intercept is $f(0) = 2(5)^2 - 4 = 46$.

v



13 **a** Since the x-intercepts are -1 and 5,

$f(x) = a(x + 1)(x - 5)$

$\therefore\ p = 5$ and $q = -1$ $\{p > q\}$

b $f(0) = -20$, so $a(1)(-5) = -20$

$\therefore\ a = 4$

c The average of the x-intercepts is $\dfrac{-1 + 5}{2} = 2$

$\therefore$ the axis of symmetry is $x = 2$.

14 **a** The graph touches the x-axis at -4, so

$f(x) = a(x + 4)^2$

$\therefore\ p = -4$

b $f(0) = -8$, so $a(4)^2 = -8$

$\therefore\ a = -\frac{1}{2}$

c The axis of symmetry is $x = -4$.

d $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.

$\therefore\ g(x) = f(-x)$

$= -\frac{1}{2}(-x + 4)^2$

$= -\frac{1}{2}(x - 4)^2$

15 **a** Since the vertex is at $(5, -12)$, the axis of symmetry is $x = 5$.

b Since the vertex is at $(5, -12)$, $f(x) = a(x - 5)^2 - 12$

$\therefore\ h = 5$ and $k = -12$.

c $f(0) = 38$, so $a(-5)^2 - 12 = 38$

$\therefore\ 25a = 50$

$\therefore\ a = 2$

16 **a** **i** When $x = 0$, $y = 9$.

$\therefore$ the y-intercept is 9.

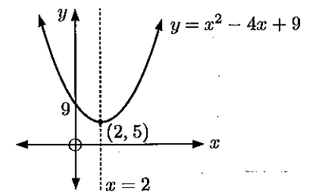
ii $y = x^2 - 4x + 9$

$= x^2 - 4x + 2^2 + 9 - 2^2$

$= (x - 2)^2 + 5$

iii The vertex is at $(2, 5)$.

iv



b **i** When $x = 0$, $y = -2$.

$\therefore$ the y-intercept is -2.

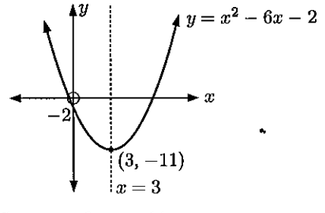
ii $y = x^2 - 6x - 2$

$= x^2 - 6x + 3^2 - 2 - 3^2$

$= (x - 3)^2 - 11$

iii The vertex is at $(3, -11)$.

iv



c **i** When $x = 0$, $y = 11$.

$\therefore$ the y-intercept is 11.

ii $y = 4x^2 + 16x + 11$

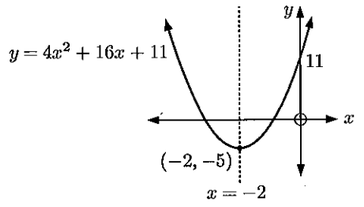
$= 4(x^2 + 4x) + 11$

$= 4(x^2 + 4x + 2^2) + 11 - 4 \times 2^2$

$= 4(x + 2)^2 - 5$

iii The vertex is at $(-2, -5)$.

iv



d **i** When $x = 0$, $y = -10$.

$\therefore$ the y-intercept is -10.

ii $y = -3x^2 + 12x - 10$

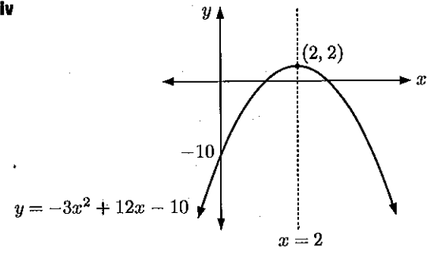
$= -3(x^2 - 4x) - 10$

$= -3(x^2 - 4x + 2^2) - 10 + 3 \times 2^2$

$= -3(x - 2)^2 + 2$

iii The vertex is at $(2, 2)$.

iv



17 **a** When $x = 0$, $y = 8$.
$\therefore$ the y-intercept is 8.

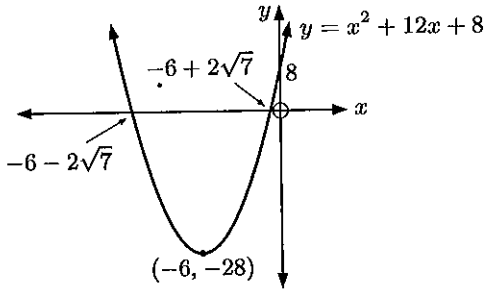
b $x^2 + 12x + 8 = 0$

$$\therefore\ x = \frac{-12 \pm \sqrt{144 - 4(1)(8)}}{2}$$
$$= \frac{-12 \pm \sqrt{112}}{2}$$
$$= \frac{-12 \pm 4\sqrt{7}}{2}$$
$$= -6 \pm 2\sqrt{7}$$

c $y = x^2 + 12x + 8$
$= x^2 + 12x + 6^2 + 8 - 6^2$
$= (x + 6)^2 - 28$
So, the vertex is at $(-6, -28)$.

d Range $= \{y \mid y \geqslant -28\}$

e



18 **a** $x^2 + 8x + k = 0$ has $\Delta = b^2 - 4ac$
$= 8^2 - 4(1)(k)$
$= 64 - 4k$

b **I** There are no real roots when $\Delta < 0$
$\therefore\ 64 - 4k < 0$
$\therefore\ 64 < 4k$
$\therefore\ k > 16$

II There are two distinct real roots when $\Delta > 0$
$\therefore\ k < 16$

19 $y = mx^2 + 4x + 6$ lies entirely above the x-axis when $m > 0$ *and* $\Delta < 0$
So, $m > 0$ *and* $4^2 - 4(m)(6) < 0$
$\therefore\ 16 - 24m < 0$
$\therefore\ -24m < -16$
$\therefore\ m > \frac{2}{3}$

So, $m > \frac{2}{3}$.

20 Since $mx^2 + (m-2)x + m = 0$ has a repeated root, $\Delta = 0$.
$\therefore\ (m-2)^2 - 4 \times m \times m = 0$
$\therefore\ -3m^2 - 4m + 4 = 0$
$\therefore\ -(3m - 2)(m + 2) = 0$
$\therefore\ m = \frac{2}{3}$ or -2

21 **a** $(-2)^2 + b(-2) + (b - 2) = 0$ {-2 is a solution}
$\therefore\ 4 - 2b + b - 2 = 0$
$\therefore\ b = 2$

b Since $b = 2$, the equation is $x^2 + 2x = 0$
$\therefore\ x(x + 2) = 0$
$\therefore\ x = 0$ is the other solution.

22 **a** Domain $= \{x \mid x \neq 1\}$, Range $= \{y \mid y \neq 1\}$

b $f(x) = \dfrac{x+2}{x-1}$ is undefined when $x = 1$, so $x = 1$ is a vertical asymptote.

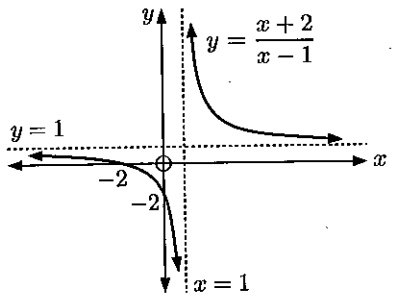
Now $f(x) = \dfrac{x+2}{x-1} = \dfrac{1 + \frac{2}{x}}{1 - \frac{1}{x}}$

$\therefore$ as $|x| \to \infty$, $f(x) \to \frac{1}{1} = 1$
$\therefore\ y = 1$ is a horizontal asymptote.

c $f(0) = \dfrac{0+2}{0-1} = -2$
So, the y-intercept is -2.

$f(x) = 0$ when $\dfrac{x+2}{x-1} = 0$
$\therefore\ x + 2 = 0$
$\therefore\ x = -2$
So, the x-intercept is -2.

d



23 **a** Domain $= \{x \mid x \neq 2\}$, Range $= \{y \mid y \neq 4\}$

b $f(x) = 4 - \dfrac{1}{x-2}$ is undefined when $x = 2$, so $x = 2$ is a vertical asymptote.

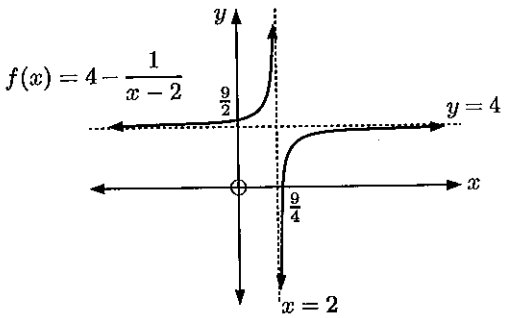
As $|x| \to \infty$, $\dfrac{1}{x-2} \to 0$, so $f(x) \to 4$.
$\therefore\ y = 4$ is a horizontal asymptote.

c $f(0) = 4 - \frac{1}{0-2} = \frac{9}{2}$
So, the y-intercept is $\frac{9}{2}$.

$f(x) = 0$ when $4 - \dfrac{1}{x-2} = 0$
$\therefore\ \dfrac{1}{x-2} = 4$
$\therefore\ x - 2 = \frac{1}{4}$
$\therefore\ x = \frac{9}{4}$

So, the x-intercept is $\frac{9}{4}$.

d



24 **a**