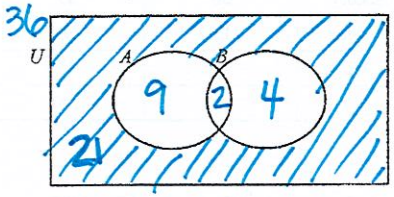
Key

Probability Practice (~62 minutes)

1. The following Venn diagram shows a sample space U and events A and B.



$n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.

(a) On the diagram, shade the region $(A \cup B)'$.

(b) Find

(i) $n(A \cap B)$: 2

(ii) $P(A \cap B)$. $\frac{2}{36} = \frac{1}{18}$

(c) Explain why events A and B are not mutually exclusive.

since $P(A \cap B) \neq 0$, A & B aren't mutually exclusive

(Total 4 marks)

2. In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.

(a) Using this information, complete the table below.

	Males	Females	Totals
Unemployed	20	40	60
Employed	90	50	140
Totals	110	90	200

(b) If a person is selected at random from this group of 200, find the probability that this person is

(i) an unemployed female; $\frac{40}{200} = \frac{1}{5}$

(ii) a male, given that the person is employed. $\frac{90}{140} = \frac{9}{14}$

(Total 4 marks)

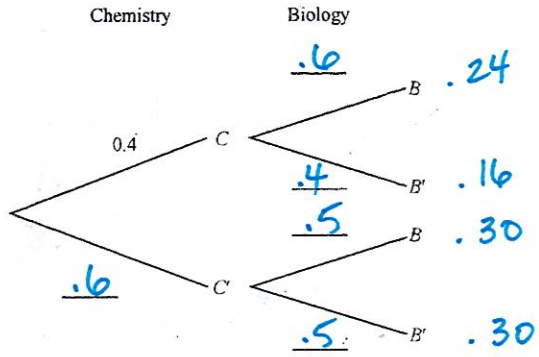
Formula

$P(M \mid E) = \frac{P(M \cap E)}{P(E)} = \frac{\frac{90}{200}}{\frac{140}{200}} = \frac{90}{140} = \frac{9}{14}$

3. The events B and C are dependent, where C is the event "a student takes Chemistry", and B is the event "a student takes Biology". It is known that

$P(C) = 0.4$, $P(B \mid C) = 0.6$, $P(B \mid C') = 0.5$.

(a) Complete the following tree diagram.

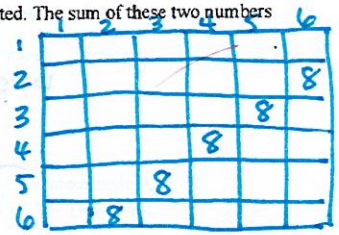


(b) Calculate the probability that a student takes Biology. .24 + .30 = .54

(c) Given that a student takes Biology, what is the probability that the student takes Chemistry?

$P(C \mid B) = \frac{P(C \cap B)}{P(B)} = \frac{.24}{.54} = .44$ **(Total 4 mar**

4. Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is S. Find the probability that



(a) S is less than 8; $\frac{21}{36} = \frac{7}{12}$

(b) at least one die shows a 3; $\frac{11}{36}$

(c) at least one die shows a 3, given that S is less than 8.

$P(\text{at least one 3} \mid S < 8)$

$\frac{\frac{7}{36}}{\frac{21}{36}} = \frac{12}{36} = \frac{1}{3}$

(Total 7 mar

5. For events A and B, the probabilities are $P(A) = \frac{3}{11}$, $P(B) = \frac{4}{11}$.

Calculate the value of $P(A \cap B)$ if

(a) $P(A \cup B) = \frac{6}{11}$; $\frac{1}{11}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{6}{11} = \frac{3}{11} + \frac{4}{11} - P(A \cap B)$

$\frac{1}{11} = P(A \cap B)$

(b) events A and B are independent.

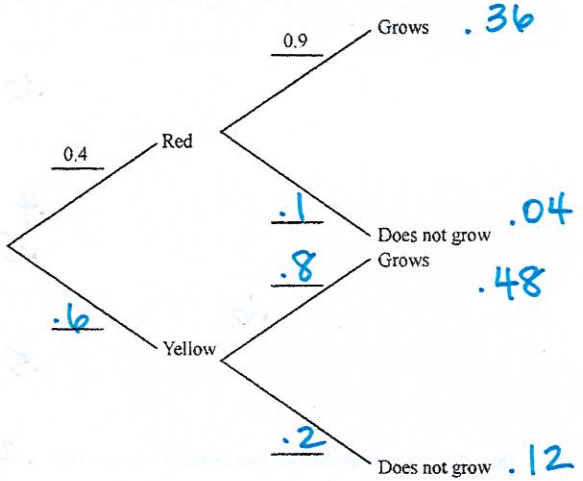
$P(A \cap B) = P(A) \cdot P(B)$

$= \frac{12}{121}$

(Total 6 mar

6. A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

(a) Complete the probability tree diagram below.



(3)

(b) (i) Calculate the probability that the chosen seed is red and grows. .36

(ii) Calculate the probability that the chosen seed grows. .48 + .36 = .84

(iii) Given that the seed grows, calculate the probability that it is red.

$P(\text{Red} \mid \text{Grows}) = \frac{P(R \cap G)}{P(G)} = \frac{.36}{.84} = .429$ (24)

(7)

(Total 10 marks)

7. A class contains 13 girls and 11 boys. The teacher randomly selects four students. Determine the probability that all four students selected are girls.

$\frac{13}{24} \cdot \frac{12}{23} \cdot \frac{11}{22} \cdot \frac{10}{21} = \frac{65}{966}$ or .067

(Total 6 marks)

8. Consider the events A and B, where $P(A) = \frac{2}{5}$, $P(B') = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{8}$.

(a) Write down $P(B)$. $1 - \frac{1}{4} = \frac{3}{4}$

(b) Find $P(A \cap B)$. $\frac{7}{8} = \frac{2}{5} + \frac{3}{4} - P(A \cap B)$

$\frac{11}{40} = P(A \cap B)$

(c) Find $P(A \mid B)$.

$\frac{P(A \cap B)}{P(B)} = \frac{\frac{11}{40}}{\frac{3}{4}} = \frac{11}{30}$

(Total 6 marks)

9. The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

(a) A student from the college is selected at random.
Let A be the event the student studies Art.
Let B be the event the student is in Year 2. → $P(B) = \frac{100}{210} = \frac{10}{21}$

(i) Find $P(A)$. $\frac{80}{210} = \frac{8}{21}$

(ii) Find the probability that the student is a Year 2 Art student. $\frac{35}{210} = \frac{1}{6}$

(iii) Are the events A and B independent? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$, if A & B are indy.
But $\frac{1}{6} \neq \frac{8}{21} \cdot \frac{10}{21}$

(b) Given that a History student is selected at random, calculate the probability that the student is in Year 1. $P(\text{Year 1} \mid \text{History})$

$\frac{50}{85} = \frac{10}{17}$

(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2.

$P(Y1 \& Y2 \text{ or } Y2 \& Y1)$

$= \frac{110}{210} \cdot \frac{100}{209} + \frac{100}{210} \cdot \frac{110}{209}$

$= \frac{200}{399}$

(Total 12 ma