IB Math SL 1 Name Key

Logs & Exponentials

1. A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\,e^{0.4t}$.

   (a) How many leopards are there after 2 years?

   (b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

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Working:

a. $N = 10e^{(.4)(2)} = 22$ leopards

b. $100 = 10e^{.4t}$

$10 = e^{.4t}$

$e^{.4t} = 10$

$\log_e 10 = .4t$

$t = 5.76$ yrs

Answers:

(a) 22 leopards

(b) 5.76 yrs

(Total 4 marks)

2. Solve the following equations.

   (a) $\ln(x + 2) = 3$.

   (b) $10^{2x} = 500$.

a. $\ln_e(x+2) = 3$

$e^3 = x + 2$

$e^3 - 2 = x$

$18.1 = x$

b. $10^{2x} = 500$

$\log_{10} 500 = 2x$

$1.35 = x$

(Total 6 marks)

3. Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

Working:

$\log_{10} x - \log_{10}(y^2 z^{1/2})$

$\log_{10} x - [\log_{10} y^2 + \log_{10} z^{1/2}]$

$\log_{10} x - 2\log_{10} y - \frac{1}{2}\log_{10} z$

Answer:

$p - 2q - \frac{1}{2}r$

or

$p - (2q + \frac{1}{2}r)$

(Total 6 marks)

4. Solve the following equations:

(a) $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$. **(4)**

(b) $4^x - 7(2^x) + 12 = 0$. **(4)**

a. $(3^2)^{x-1} = (3^{-1})^{2x}$

$3^{2x-2} = 3^{-2x}$

$2x - 2 = -2x$

$4x = 2$

$x = \frac{1}{2}$

b. $(2^x)^2 - 7(2^x) + 12 = 0$

$w^2 - 7w + 12 = 0$

$(w-4)(w-3) = 0$

$(2^x - 4)(2^x - 3) = 0$

$2^x - 4 = 0$ $\quad 2^x - 3 = 0$

$2^x = 4$ $\quad 2^x = 3$

$x = 2$ $\quad \log_2 3 = x$

$1.58 = x$

(Total 8 marks)