Example 17

Find the mean and the standard deviation of a normally distributed random variable X, if $P(x \geqslant 50) = 0.2$ and $P(x \leqslant 20) = 0.3$.

$P(x \leqslant 20) = 0.3$

$\therefore \ P(z \leqslant \dfrac{20-\mu}{\sigma}) = 0.3$

$\therefore \ \dfrac{20-\mu}{\sigma} = \text{invNorm}(0.3)$

$\therefore \ \dfrac{20-\mu}{\sigma} \doteqdot -0.5244$

$\therefore \ 20 - \mu \doteqdot -0.5244\sigma$ ..... (1)

$P(x \geqslant 50) = 0.2$

$\therefore \ P(x \leqslant 50) = 0.8$

$\therefore \ P(z \leqslant \dfrac{50-\mu}{\sigma}) = 0.8$

$\therefore \ \dfrac{50-\mu}{\sigma} = 0.8416$

$\therefore \ 50 - \mu \doteqdot 0.8416\sigma$ ..... (2)

Solving (1) and (2) simultaneously we get $\mu \doteqdot 31.5$, $\sigma \doteqdot 22.0$ Check with a GC!

6 **a** A random variable, X, is normally distributed. Find the mean and the standard deviation of X, given that $P(X \geqslant 80) = 0.1$ and $P(X \leqslant 30) = 0.15$.

b It was found that 10% of the students scored at least 80 marks and no more than 15% scored less than 30 marks in the Mathematics examination at the end of the year. What proportion of students scored more than 50 marks?

7 Circular metal tokens are used to operate a washing machine in a laundromat. The diameters of the tokens are known to be normally distributed. Only tokens with diameters between 1.94 and 2.06 cm will operate the machine.

a Find the mean and the standard deviation of the distribution given that 2% of the tokens are too small, and 3% are too large.

b Find the probability that less than two tokens out of a batch of 20 will not operate the machine.

8 A student scored 70 for a Science exam and 66 for a Geography exam. If the class scores are normally distributed with a mean and a standard deviation for Science of 60 and 10 and for Geography 50 and 12, in which subject did the student achieve a higher standard, and what percentage of others achieved lower marks in each subject?

REVIEW SET 29A

1 $P(x) = \dfrac{a}{x^2+1}$ for $x = 0, 1, 2, 3$ is a probability distribution function.

a Find a. **b** Hence, find $P(x \geqslant 1)$.

2 A random variable X has probability distribution function $P(x) = C_x^4 \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$ for $x = 0, 1, 2, 3, 4$.

a Find $P(x)$ for $x = 0, 1, 2, 3, 4$. **b** Find μ and σ for this distribution.

3 A manufacturer finds that 18% of the items produced from one of the assembly lines are defective. During a floor inspection, the manufacturer randomly selects ten items. Find the probability that the manufacturer finds:

a one defective **b** two defective **c** at least two defectives.

4 A random sample of 120 toothbrushes is made (with replacement) from a very large batch where 4% are known to be defective. Find:

a the mean

b the standard deviation of the number of defectives in the sample.

5 At a social club function, a dice game is played where on a single roll of a six-sided die the following payouts are made:
\$2 for an odd number, \$3 for a 2, \$6 for a 4 and \$9 for a 6.

a What is the expected return for a single roll of the die?

b If the club charges \$5 for each roll, how much money would the club expect to make if 75 people played the game once each?

6 Lakshmi rolls a normal six-sided die and wins twice the number of dollars as the number shown on the face.

a How much does Lakshmi expect to win from one roll of the die?

b If it costs \$8 to play the game, would you advise Lakshmi to play several games? Explain your answer.

7 The arm lengths of 18 year old females are normally distributed with mean 64 cm and standard deviation 4 cm.

a Find the percentage of 18 year old females whose arm lengths are:

i between 60 cm and 72 cm **ii** greater than 60 cm.

b Find the probability that a randomly chosen 18 year old female has an arm length in the range 56 cm to 68 cm.

8 The length of steel rods produced by a machine is normally distributed with a standard deviation of 3 mm. It is found that 2% of all rods are less than 25 mm long. Find the mean length of rods produced by the machine.

REVIEW SET 29B

1 A discrete random variable X has probability distribution function $P(x)$ where $P(x) = k\left(\frac{3}{4}\right)^x\left(\frac{1}{4}\right)^{3-x}$ where $x = 0, 1, 2, 3$ and k is a constant.

a Find k. **b** Find $P(x \geqslant 1)$.

2 An insurance company covers a \$45 000 painting against fire, theft and accidental damage. If the painting is destroyed by fire the policy is paid out in full. If it is stolen the company will pay \$30 000 and if accidentally damaged \$10 000. From past experience the company knows that the probabilities of fire, theft and accidental damage are 0.000 72, 0.0023 and 0.0088 respectively. How much should the company charge to cover the painting if they want a \$250 expected return?

3 A pistol shooter has a probability of 0.96 of hitting a target with each shot. If she fires four times at a target, find the probability that:

a all four shots hit the target **b** she does not hit the target

c at least three shots hit the target **d** only one shot hits the target.

4 Suppose X is Bin(4, p).

a Find $P(0)$, $P(1)$, $P(2)$, $P(3)$ and $P(4)$ using $P(x) = C_x^4 p^x q^{4-x}$ and tabulate the results in:

x_i	0	1	2	3	4
p_i					

b Use $\mu = \sum p_i x_i$ to show that $\mu = 4p$.

c Use $\sigma^2 = \sum x_i^2 p_i - \mu^2$ to show that $\sigma = \sqrt{4pq}$. [**Reminder**: $q = 1 - p$]

5 The contents of a certain brand of soft drink can is normally distributed with mean 377 mL and standard deviation 4.2 mL.

a Find the percentage of cans with contents:

i less than 368.6 mL **ii** between 372.8 mL and 389.6 mL

b Find the probability that a randomly selected can has contents between 364.4 mL and 381.2 mL.

6 The life of a Xenon battery has a normal distribution. The mean is 33.2 weeks and the standard deviation is 2.8 weeks. If a battery is selected at random,

a find the probability that it will last at least 35 weeks.

b Find the maximum number of weeks for which the manufacturer can expect that not more than 8% of batteries will fail.

REVIEW SET 29C

1 A random variable X has probability distribution function given by:

x	0	1	2	3	4
$P(x)$	0.10	0.30	0.45	0.10	k

a Find k.

b Find the mean μ, and standard deviation σ, for the distribution of X.

2 Only 40% of young trees that are planted will survive the first year. The Botanical Gardens buys five young trees. Assuming independence, calculate the probability that during the first year:

a exactly one tree will survive

b at most one tree will survive

c at least one tree will survive.

3 From data over the last fifteen years it is known that the chance of a netballer needing major knee surgery in any one season is 0.0132. In 1998 there were 487 cases in which this happened. Find the mean and standard deviation of the number of cases.

4 The edible part of a batch of Coffin Bay oysters is normally distributed with mean 38.6 grams and standard deviation 6.3 grams. Given that the random variable X is the mass of a Coffin Bay oyster:

a find a if $P(38.6 - a \leqslant x \leqslant 38.6 + a) = 0.6826$

b find b if $P(x \geqslant b) = 0.8413$.

5 Staplers are manufactured for \$5.00 each and are sold for \$20.00 each. The staplers have a money-back guarantee to last three years. The mean life is actually 3.42 years and the standard deviation is 0.4 years. If the life of these staplers is normally distributed, how much profit would we expect from selling all of a batch of 2000?

6 A random variable X has probability distribution function $f(x) = ax^2(2 - x)$ for $0 < x < 2$.

a Show that $a = \frac{3}{4}$. (**Hint**: What is $\int_0^2 f(x)\,dx$ equal to?)

b Find the mode of X.

c Find the median of X.

d Find $P(0.6 < x < 1.2)$.