74 Marks

Key

Normal Distribution Practice

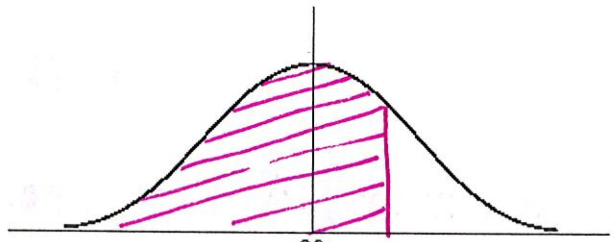
1. A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $P(X \leq 24.5)$. normalcdf (-9999, 24.5, 20, 3) = .933

(3)

(b) Let $P(X \leq k) = 0.85$.

(i) Represent this information on the following diagram.

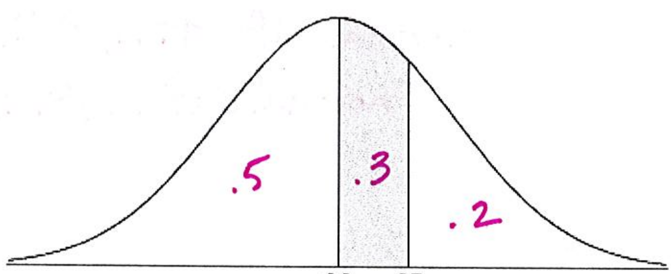


(ii) Find the value of k.

invNorm (.85, 20, 3) = 23.1

(5)
(Total 8 marks)

2. Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find $P(X > 27)$. = .2

(2)

(b) Find the standard deviation of X.

from part a, $P(x < 27) = .8$

invNorm (.8, 0, 1) = .842 (z-score)

$.842 = \frac{27-25}{\sigma}$, $\sigma = 2.38$

(5)
(Total 7 marks)

3. A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \leq 475)$. normalcdf (-9999, 475, 450, 20) = .894

(2)

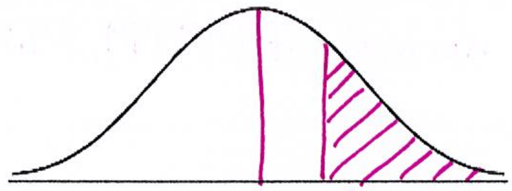
(b) Given that $P(X > a) = 0.27$, find a.

$P(X < a) = .73$ invNorm (.73, 450, 20) = 462

(4)
(Total 6 marks)

4. Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

(a) On the diagram below, shade the region representing $P(X > 105)$.



(2)

(b) Given that $P(X < d) = P(X > 105)$, find the value of d. $d = 95$

(2)

(c) Given that $P(X > 105) = 0.16$ (correct to two significant figures), find $P(d < X < 105)$.

normalcdf(95, 105, 100, 5) = .68

(2)

(Total 6 marks)

5. A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

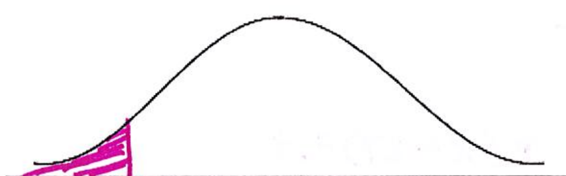
(i) weighs less than 8 g; normalcdf(-9999, 8, 7, .5) = .977

(ii) weighs between 6 g and 8 g. normalcdf(6, 8, 7, .5) = .954

(4)

(b) Five percent of the biscuits in the box weigh less than d grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



(ii) Find the value of d. invNorm(.05, 7, .5) = 6.18

(5)

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g.

Find the value of μ.

$P(X < 5) = .20$

invNorm(.20, 0, 1) = -.842 (z-score)

$-.842 = \frac{5 - \mu}{.5}$

$5.42 = \mu$

(4)

(Total 13 marks)

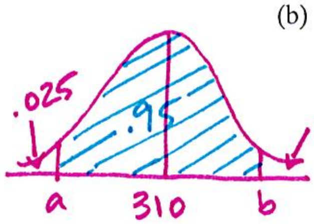
6. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more. **(2)**

normal cdf (350, 9999, 310, 30) = .091

(b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b. **(3)**

(Total 5 marks)



$P(x<a) = .025$
invNorm (.025, 310, 30)
= 251
so $a = 251$

$P(x<b) = .975$
invNorm(.975, 310, 30)
= 369
so $b = 369$

7. The heights of certain flowers follow a normal distribution. It is known that 20% of these flowers have a height less than 3 cm and 10% have a height greater than 8 cm.

$\to P(x<3) = .20$

$\to P(x>8) = .10$

Find the value of the mean μ and the standard deviation σ.

$\to$ or $P(x<8) = .90$

(Total 6 marks)

invNorm (.20, 0, 1) = −.842 $\to$ $-.842 = \frac{3-\mu}{\sigma}$

invNorm (.90, 0, 1) = 1.282

$1.282 = \frac{8-\mu}{\sigma}$

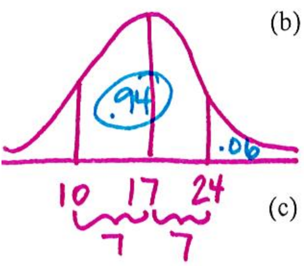
$8-\mu = 1.282\sigma$
$3-\mu = -.842\sigma$
$5 = 2.124\sigma$
$\sigma = 2.35$, $\mu = 4.98$

8. The heights of trees in a forest are normally distributed with mean height 17 metres. One tree is selected at random. The probability that a selected tree has a height greater than 24 metres is 0.06. $\to P(x>24) = .06$

$P(x<24) = .94$

(a) Find the probability that the tree selected has a height less than 24 metres. **(2)**

$P(x<24) = 1 - .06 = .94$



(b) The probability that the tree has a height less than D metres is 0.06. Find the value of D. **(3)**

$P(x<D) = .06$
$D = 10$

(c) A woodcutter randomly selects 200 trees. Find the expected number of trees whose height lies between 17 metres and 24 metres. **(4)**

(Total 9 marks)

$\frac{1-2(.06)}{2} = \frac{.88}{2} \leftarrow P(17<x<24)$

$E(x) = 200(.44)$
$= 88$

9. The scores of a test given to students are normally distributed with a mean of 21.
80 % of the students have scores less than 23.7. $P(x < 23.7) = .80$

(a) Find the standard deviation of the scores. **(3)**

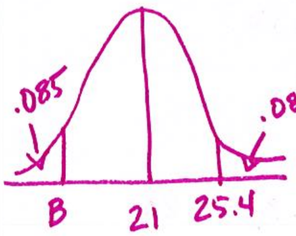
$\text{invNorm}(.80, 0, 1) = .842$ (z-score)

$.842 = \dfrac{23.7-21}{\sigma}$, $\sigma = 3.21$

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than *b*.

(b) (i) Find the probability the student has a score less than 25.4.

$\text{normalcdf}(-9999, 25.4, 21, 3.21) = .915$



(ii) Find the value of *b*.

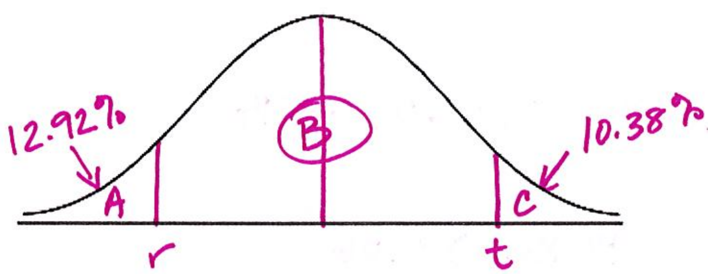
$\text{invNorm}(.085, 21, 3.21) = 16.6$

(4)
(Total 7 marks)

10. The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A.
The tallest 10.38% are in category C.
All the other plants are in category B with heights between *r* cm and *t* cm.

(a) Complete the following diagram to represent this information.



(2)

(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of *r* and of *t*. **(5)**
(Total 7 marks)

$P(x < r) = .1292$

$\text{invNorm}(.1292, 6.84, .25)$

$r = 6.56$

- - - - - - - - - - - - - - -

$P(x > t) = .1038$

so $P(x < t) = 1 - .1038 = .8962$

$\text{invNorm}(.8962, 6.84, .25)$

$t = 7.16$