Midterm Review #2

Sequences and Series

1. The *Ăcme* insurance company sells two savings plans, Plan A and Plan B.
   For Plan A, an investor starts with an initial deposit of \$1000 and increases this by \$80 each month, so that in the second month, the deposit is \$1080, the next month it is \$1160, and so on.
   For Plan B, the investor again starts with \$1000 and each month deposits 6% more than the previous month.
   a) Write down the amount of money invested under Plan B in the second and third months.
   Give your answers to parts (b) and (c) correct to the nearest dollar.
   b) Find the amount of the 12th deposit for each Plan.
   c) Find the total amount of money invested during the first 12 months
      i) under Plan A;
      ii) under Plan B.

2. **(no calculator)** Given that 24, b, c, are the first three terms of an arithmetic sequence, with non-zero common difference, and that 24, c, b, are the first three terms of a geometric sequence, find b and c.

3. **(no calculator)** In an arithmetic sequence, the first term is –2, the fourth term is 16, and the n^{th} term is 11 998.
   a) Find the common difference d.
   b) Find the value of n.

Exponents and Logarithms

4. Solve for real x:

   a) $\log_8 x = 3^{-1}$ b) $8^{-x} = \left(\frac{1}{4}\right)^3$ c) $\log_{27} x = 1 - \log_{27}(x - 0.4)$ d) $2^x = 7^{x-1}$

   e) $\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9 x$

5. The number of radioactive atoms N of a particular material present at time t years may be written in the form $N = 5000\, e^{-kt}$, where 5000 is the number of atoms present when $t = 0$, and k is a positive constant. It is found that $N = 2500$ when $t = 5$ years.
   a) Determine the value of k.
   b) At what value of t will $N = 50$?

Binomial Theorem (Pascal's triangle, combinations)

6. In one of the terms in the expansion of $(x^3 - 3y^2)^5$, the powers of x and y will be identical. Find this term, giving your answer in its simplest form.

7. **(no calculator)** Find the coefficient of y^3 in the expansion of $(3 - 2y)^5$, simplifying your answer as much as possible.

8. Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.
   a) How many terms are there in this expansion?
   b) Find the constant term in this expansion.

Functions and Graphing

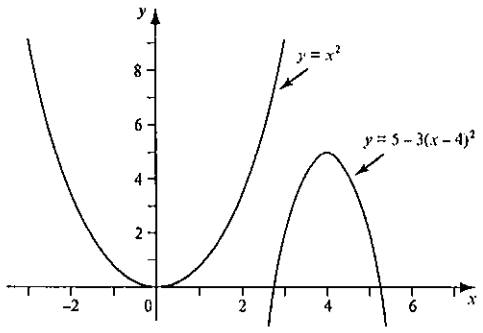
9. **(no calculator)** Let $f(x) = 2^x$, and $g(x) = \frac{x}{x-2}$, $(x \neq 2)$. Find
   a) $(g \circ f)(3)$; b) $g^{-1}(5)$.

10. **(no calculator)** The diagram shows parts of the graphs of $y = x^2$ and $y = 5 - 3(x-4)^2$.
The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x-4)^2$ by these transformations.

A reflection in the line $y = 0$ **followed by**
a vertical stretch with scale factor k **followed by**
a horizontal translation of p units **followed by**
a vertical translation of q units.

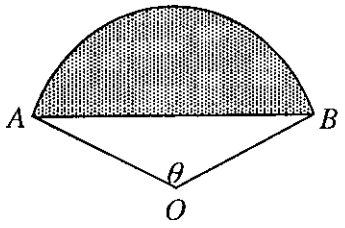
Write down the values of

a) k; b) p; c) q.

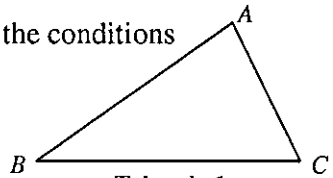


Trig

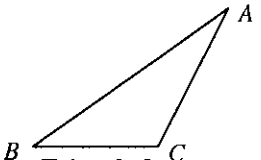
1. **(no calculator)** The diagram at right shows a sector AOB of a circle of radius 15 cm and centre O. The angle θ at the centre of the circle is 2 radians.
   a) Calculate the length of arc AB.
   b) Calculate the area of the sector AOB.
   c) Calculate the area of the shaded region.
   d) Calculate the perimeter of the shaded region.



2. The diagrams show two triangles both satisfying the conditions $AB = 20$ cm, $AC = 17$ cm, $A\hat{B}C = 50°$.
Calculate the size of $A\hat{C}B$ in **Triangle 2**.

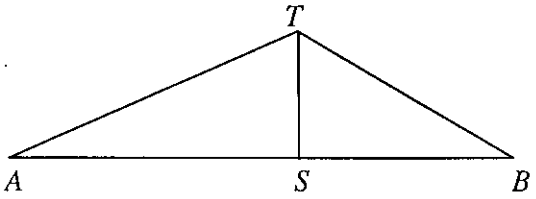


Triangle 1



Triangle 2

3. S is the base of a vertical pole TS.
S lies on AB, where A and B are 92.5 meters apart on horizontal ground.
$\angle TAB = 20°$ and $\angle TBA = 30°$.
Calculate the length of the pole TS to the nearest tenth of a meter.



4. In the triangle ABC it is given that $BC = 9$ cm, $CA = 13$ cm, $AB = 10$ cm and D is the midpoint of $[AB]$. By applying the cosine formula to each of two triangles, or otherwise, find CD.