SECTION A

1. (a) $d = 3$ ***A1*** ***N1***

[1 mark]

(b) (i) correct substitution into term formula ***(A1)***

eg $u_{100} = 5 + 3(99)$, $5 + 3(100 - 1)$

$u_{100} = 302$ ***A1*** ***N2***

(ii) correct substitution into sum formula ***(A1)***

eg $S_{100} = \frac{100}{2}\left(2(5) + 99(3)\right)$, $S_{100} = \frac{100}{2}(5 + 302)$

$S_{100} = 15350$ ***A1*** ***N2***

[4 marks]

(c) correct substitution into term formula ***(A1)***

eg $1502 = 5 + 3(n - 1), 1502 = 3n + 2$

$n = 500$ ***A1*** ***N2***

[2 marks]

Total [7 marks]

2. (a) valid approach ***(M1)***

eg $35 - 26$, $26 + p = 35$

$p = 9$ ***A1*** ***N2***

[2 marks]

(b) (i) mean $= 26.7$ ***A2*** ***N2***

(ii) recognizing that variance is $(\text{sd})^2$ ***(M1)***

eg $11.021\ldots^2$, $\sigma = \sqrt{\text{var}}$, $11.158\ldots^2$

$\sigma^2 = 121$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

3. (a) $p = 5$, $q = 7$, $r = 7$ (accept $r = 5$) ***A1A1A1*** ***N3***

[3 marks]

(b) correct working ***(A1)***

eg $\binom{12}{7} \times (3x)^5 \times (-2)^7$, 792, 243, -2^7, 24634368

coefficient of term in x^5 is -24634368 ***A1*** ***N2***

Note: Do not award the final ***A1*** for an answer that contains x.

[2 marks]

Total [5 marks]

4. (a) (i) $\boldsymbol{A} = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 1 & 0 \\ -2 & -1 & 2 \end{pmatrix}$ ***A1*** ***N1***

(ii) $\boldsymbol{A}^{-1} = \begin{pmatrix} 2 & 1 & -1 \\ -2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ ***A2*** ***N2***

Note: Award ***A1*** for 6, 7 or 8 correct elements.

[3 marks]

(b) evidence of multiplying by $\boldsymbol{A}^{-1}$ (in any order) ***(M1)***

eg $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$, $\boldsymbol{B}\boldsymbol{A}^{-1}$, one correct element

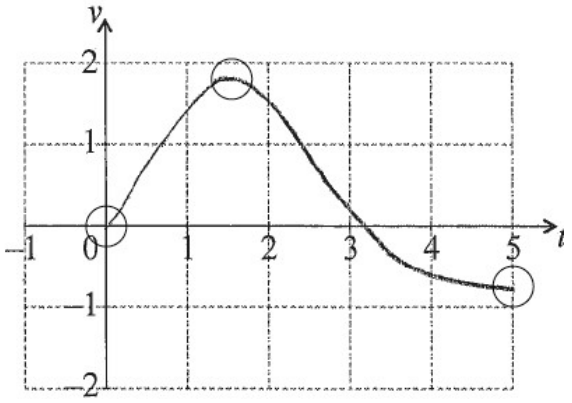
$\boldsymbol{X} = \begin{pmatrix} 9 \\ -8 \\ 3.5 \end{pmatrix}$ (accept $x = 9$, $y = -8$, $z = 3.5$) ***A2*** ***N3***

Note: Award ***A1*** for two correct elements.

[3 marks]

Total [6 marks]

5. (a)



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape crossing x-axis with $3 < x < 3.5$.
Only if this ***A1*** is awarded, award the following:
A1 for maximum in circle, ***A1*** for endpoints in circle.

[3 marks]

(b) (i) $t = \pi (\text{exact}), 3.14$ ***A1*** ***N1***

(ii) recognizing distance is area under velocity curve ***(M1)***
eg $s = \int v$, shading on diagram, attempt to integrate v

valid approach to find the total area ***(M1)***
eg area A + area B, $\int v\mathrm{d}t - \int v\mathrm{d}t$, $\int_0^{3.14} v\mathrm{d}t + \int_{3.14}^{5} v\mathrm{d}t$, $\int |v|$

correct working with integration and limits (accept dx or missing dt) ***(A1)***
eg $\int_0^{3.14} v\mathrm{d}t + \int_5^{3.14} v\mathrm{d}t$, $3.067\ldots + 0.878\ldots$, $\int_0^5 \left|\mathrm{e}^{\sin t} - 1\right|$

distance $= 3.95\,(\text{m})$ ***A1*** ***N3***

[5 marks]
Total [8 marks]

6. (a) (i) $k = 2$ ***A1*** ***N1***

(ii) $p = -1$ ***A1*** ***N1***

(iii) $q = 5$ ***A1*** ***N1***

[3 marks]

(b) recognizing one transformation ***(M1)***
eg horizontal stretch by $\frac{1}{3}$, reflection in x-axis,

A′ is $(2, -5)$ ***A1A1*** ***N3***

[3 marks]

Total [6 marks]

7. recognizing one quartile probability (may be seen in a sketch) ***(M1)***
eg $\mathrm{P}(X < Q_3) = 0.75$, 0.25

finding standardized value for either quartile ***(A1)***
eg $z = 0.67448\ldots$, $z = -0.67448\ldots$

attempt to set up equation (must be with z – values) ***(M1)***
eg $0.67 = \frac{Q_3 - 150}{10}$, $-0.67448 = \frac{x - 150}{10}$

one correct quartile
eg $Q_3 = 156.74\ldots$, $Q_1 = 143.25\ldots$ ***(A1)***

correct working ***(A1)***
eg other correct quartile, $Q_3 - \mu = 6.744\ldots$

valid approach for IQR (seen anywhere) ***(A1)***
eg $Q_3 - Q_1$, $2(Q_3 - \mu)$

$\mathrm{IQR} = 13.5$ ***A1*** ***N4***

[7 marks]

SECTION B

8. (a) evidence of choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$, $\text{CD}^2 + \text{AD}^2 - 2 \times \text{CD} \times \text{AD}\cos D$

correct substitution ***A1***

eg $11.5^2 + 8^2 - 2 \times 11.5 \times 8\cos 104$, $196.25 - 184\cos 104$

$\text{AC} = 15.5\,(\text{m})$ ***A1*** ***N2***

[3 marks]

(b) (i) **METHOD 1**

evidence of choosing sine rule ***(M1)***

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\sin \text{A}\hat{\text{C}}\text{D}}{\text{AD}} = \frac{\sin D}{\text{AC}}$

correct substitution ***A1***

eg $\frac{\sin \text{A}\hat{\text{C}}\text{D}}{8} = \frac{\sin 104}{15.516\ldots}$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ ***A1*** ***N2***

METHOD 2

evidence of choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

eg $8^2 = 11.5^2 + 15.516\ldots^2 - 2(11.5)(15.516\ldots)\cos C$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ ***A1*** ***N2***

(ii) subtracting **their** $\text{A}\hat{\text{C}}\text{D}$ from 73 ***(M1)***

eg $73 - \text{A}\hat{\text{C}}\text{D}$, $70 - 30.017...$

$\text{A}\hat{\text{C}}\text{B} = 43.0^\circ$ ***A1*** ***N2***

[5 marks]

(c) correct substitution ***(A1)***

eg $\text{area } \Delta\text{ADC} = \frac{1}{2}(8)(11.5)\sin 104$

$\text{area} = 44.6\ (\text{m}^2)$ ***A1*** ***N2***

[2 marks]

(d) attempt to subtract ***(M1)***

eg $\text{circle} - \text{ABCD}$, $\pi r^2 - \Delta\text{ADC} - \Delta\text{ACB}$

$\text{area } \Delta\text{ACB} = \frac{1}{2}(15.516\ldots)(14)\sin 42.98\ (= 74.0517\ldots)$ ***(A1)***

correct working ***A1***

eg $\pi(8)^2 - 44.6336\ldots - \frac{1}{2}(15.516\ldots)(14)\sin 42.98$, $64\pi - 44.6 - 74.1$

shaded area is $82.4\ (\text{m}^2)$ ***A1*** ***N3***

[4 marks]

Total [14 marks]

9. (a) $f(0) = \frac{100}{51}$ (exact), 1.96 ***A1*** ***N1***

[1 mark]

(b) setting up equation ***(M1)***

eg $95 = \frac{100}{1+50e^{-0.2x}}$, sketch of graph with horizontal line at $y = 95$

$x = 34.3$ ***A1*** ***N2***

[2 marks]

(c) upper bound of y is 100 ***(A1)***
lower bound of y is 0 ***(A1)***

range is $0 < y < 100$ ***A1*** ***N3***

[3 marks]

(d) **METHOD 1**

setting function ready to apply the chain rule ***(M1)***

eg $100\left(1+50e^{-0.2x}\right)^{-1}$

evidence of correct differentiation (must be substituted into chain rule) ***(A1)(A1)***

eg $u' = -100\left(1+50e^{-0.2x}\right)^{-2}$, $v' = \left(50e^{-0.2x}\right)(-0.2)$

correct chain rule derivative ***A1***

eg $f'(x) = -100\left(1+50e^{-0.2x}\right)^{-2}\left(50e^{-0.2x}\right)(-0.2)$

correct working clearly leading to the required answer ***A1***

eg $f'(x) = 1000e^{-0.2x}\left(1+50e^{-0.2x}\right)^{-2}$

$f'(x) = \frac{1000e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$ ***AG*** ***N0***

METHOD 2

attempt to apply the quotient rule (accept reversed numerator terms) ***(M1)***

eg $\frac{vu'-uv'}{v^2}$, $\frac{uv'-vu'}{v^2}$

evidence of correct differentiation inside the quotient rule ***(A1)(A1)***

eg $f'(x) = \frac{\left(1+50e^{-0.2x}\right)(0)-100\left(50e^{-0.2x}\times -0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$, $\frac{100(-10)e^{-0.2x}-0}{\left(1+50e^{-0.2x}\right)^2}$

any correct expression for derivative (0 may not be explicitly seen) ***(A1)***

eg $\frac{-100\left(50e^{-0.2x}\times -0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$

correct working clearly leading to the required answer ***A1***

eg $f'(x) = \frac{0-100(-10)e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$, $\frac{-100(-10)e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$

$f'(x) = \frac{1000e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$ ***AG*** ***N0***

[5 marks]

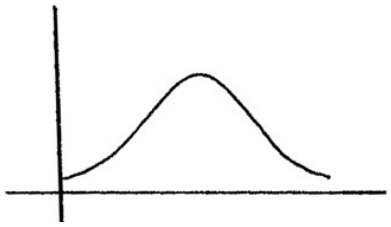
continued ...

Question 9 continued

(e) **METHOD 1**

sketch of $f'(x)$ ***(A1)***

eg



recognizing maximum on $f'(x)$ ***(M1)***

eg dot on max of sketch

finding maximum on graph of $f'(x)$ ***A1***

eg $(19.6, 5)$, $x = 19.560\ldots$

maximum rate of increase is 5 ***A1*** ***N2***

[4 marks]

METHOD 2

recognizing $f''(x) = 0$ ***(M1)***

finding any correct expression for $f''(x)$ ***(A1)***

eg $\dfrac{\left(1+50e^{-0.2x}\right)^2\left(-200e^{-0.2x}\right)-\left(1000e^{-0.2x}\right)\left(2\left(1+50e^{-0.2x}\right)\left(-10e^{-0.2x}\right)\right)}{\left(1+50e^{-0.2x}\right)^4}$

finding $x = 19.560\ldots$ ***A1***

maximum rate of increase is 5 ***A1*** ***N2***

[4 marks]

Total [15 marks]

10. (a) valid approach ***(M1)***
eg $13 + \text{diameter},\ 13 + 122$

maximum height $= 135$ (m) ***A1*** ***N2***
[2 marks]

(b) (i) $\text{period} = \frac{60}{2.4}$ ***A1***

$\text{period} = 25$ (minutes) ***AG*** ***N0***

(ii) $b = \frac{2\pi}{25}$ $(= 0.08\pi)$ ***A1*** ***N1***
[2 marks]

(c) **METHOD 1**

valid approach ***(M1)***
eg $\max -74,\ |a| = \frac{135-13}{2},\ 74-13$

$|a| = 61$ (accept $a = 61$) ***(A1)***

$a = -61$ ***A1*** ***N2***
[3 marks]

METHOD 2

attempt to substitute valid point into equation for h ***(M1)***
eg $135 = 74 + a\cos\left(\frac{2\pi \times 12.5}{25}\right)$

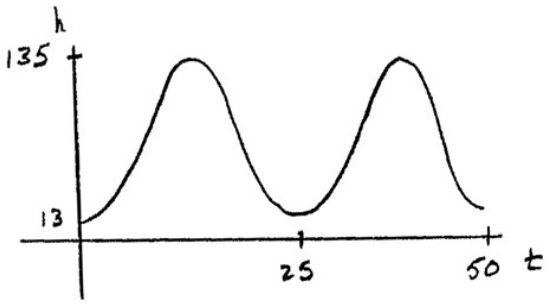
correct equation ***(A1)***
eg $135 = 74 + a\cos(\pi),\ 13 = 74 + a$

$a = -61$ ***A1*** ***N2***
[3 marks]

continued ...

Question 10 continued

(d)



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range, ***A1*** for approximately correct sinusoidal shape with 2 cycles.
Only if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions.

[4 marks]

(e) setting up inequality (accept equation) ***(M1)***
eg $h > 105$, $105 = 74 + a\cos bt$, sketch of graph with line $y = 105$

any **two** correct values for t (seen anywhere) ***A1A1***
eg $t = 8.371\ldots$, $t = 16.628\ldots$, $t = 33.371\ldots$, $t = 41.628\ldots$,

valid approach ***M1***
eg $\frac{16.628 - 8.371}{25}, \frac{t_1 - t_2}{25}, \frac{2 \times 8.257}{50}, \frac{2(12.5 - 8.371)}{25}$

$p = 0.330$ ***A1*** ***N2***

[5 marks]

Total [16 marks]