1. (a) (i) $2\boldsymbol{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ ***(A1)***

correct expression for $2\boldsymbol{a} + \boldsymbol{b}$ ***A1*** ***N2***

eg $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, $(5, -2)$, $5\boldsymbol{i} - 2\boldsymbol{j}$

(ii) correct substitution into length formula ***(A1)***

eg $\sqrt{5^2 + 2^2}$, $\sqrt{5^2 + -2^2}$

$|2\boldsymbol{a} + \boldsymbol{b}| = \sqrt{29}$ ***A1*** ***N2***

[4 marks]

(b) valid approach ***(M1)***

eg $\boldsymbol{c} = -(2\boldsymbol{a} + \boldsymbol{b})$, $5 + x = 0, -2 + y = 0$

$\boldsymbol{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) $x = 1$, $x = -3$ $\left(\text{accept } (1, 0), (-3, 0)\right)$ ***A1A1*** ***N2***

[2 marks]

(b) **METHOD 1**

attempt to find x-coordinate ***(M1)***

eg $\frac{1 + -3}{2}$, $x = \frac{-b}{2a}$, $f'(x) = 0$

correct value, $x = -1$ (may be seen as a coordinate in the answer) ***A1***

attempt to find **their** y-coordinate ***(M1)***

eg $f(-1)$, -2×2, $y = \frac{-D}{4a}$

$y = -4$ ***A1***

vertex $(-1, -4)$ ***N3***

[4 marks]

METHOD 2

attempt to complete the square ***(M1)***

eg $x^2 + 2x + 1 - 1 - 3$

attempt to put into vertex form ***(M1)***

eg $(x + 1)^2 - 4$, $(x - 1)^2 + 4$

vertex $(-1, -4)$ ***A1A1*** ***N3***

[4 marks]

Total [6 marks]

3. (a) evidence of choosing product rule *(M1)*

eg $uv' + vu'$

correct derivatives (must be seen in the product rule) $\cos x$, $2x$ *(A1)(A1)*

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1*** ***N4***

[4 marks]

(b) substituting $\frac{\pi}{2}$ into **their** $f'(x)$ *(M1)*

eg $f'\left(\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$ seen in $f'(x)$ *(A1)*

eg $0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

4. (a) attempt to solve for $\boldsymbol{X}$ *(M1)*

eg $\boldsymbol{XA} = \boldsymbol{C} - \boldsymbol{B}$, $\boldsymbol{X} + \boldsymbol{B} = \boldsymbol{CA}^{-1}$, $\boldsymbol{A}^{-1}(\boldsymbol{C} - \boldsymbol{B})$, $\boldsymbol{A}^{-1}\boldsymbol{C} - \boldsymbol{B}$

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1} \quad (= \boldsymbol{CA}^{-1} - \boldsymbol{BA}^{-1})$ *A1* *N2*

[2 marks]

(b) **METHOD 1**

$\boldsymbol{C} - \boldsymbol{B} = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$ (seen anywhere) *A1*

correct substitution into formula for 2×2 inverse *A1*

eg $\boldsymbol{A}^{-1} = \frac{1}{4-6}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$, $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

attempt to multiply $(\boldsymbol{C} - \boldsymbol{B})$ and $\boldsymbol{A}^{-1}$ (in any order) *(M1)*

eg $\begin{pmatrix} -2+3 & 1-1 \\ 8+3 & -4-1 \end{pmatrix}$, $\begin{pmatrix} 4-6 & -2+2 \\ -16-6 & 8+2 \end{pmatrix}$, two correct elements

$\boldsymbol{X} = \begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$ *A2* *N3*

Note: Award ***A1*** for three correct elements.

[5 marks]

METHOD 2

correct substitution into formula for 2×2 inverse *A1*

eg $\boldsymbol{A}^{-1} = \frac{1}{4-6}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$, $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

attempt to multiply either $\boldsymbol{BA}^{-1}$ or $\boldsymbol{CA}^{-1}$ (in any order) *(M1)*

eg $\frac{-1}{2}\begin{pmatrix} 0-3 & 0+1 \\ 4-6 & -2+2 \end{pmatrix}$, $\frac{-1}{2}\begin{pmatrix} -2-3 & -6+4 \\ \frac{3}{2}+\frac{3}{2} & \frac{9}{2}-2 \end{pmatrix}$, two correct entries

one correct multiplication *A1*

eg $\frac{-1}{2}\begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix}$, $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 12 & -5 \end{pmatrix}$

$\boldsymbol{X} = \begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$ *A2* *N3*

Note: Award ***A1*** for three correct elements. .

[5 marks]

Total [7 marks]

5. (a) **METHOD 1**

attempt to set up equation *(M1)*

eg $2=\sqrt{y-5}$, $2=\sqrt{x-5}$

correct working *(A1)*

eg $4=y-5$, $x=2^2+5$

$f^{-1}(2)=9$ ***A1*** ***N2***

[3 marks]

METHOD 2

interchanging x and y (seen anywhere) *(M1)*

eg $x=\sqrt{y-5}$

correct working *(A1)*

eg $x^2=y-5$, $y=x^2+5$

$f^{-1}(2)=9$ ***A1*** ***N2***

[3 marks]

(b) recognizing $g^{-1}(3)=30$ *(M1)*

eg $f(30)$

correct working *(A1)*

eg $(f\circ g^{-1})(3)=\sqrt{30-5}$, $\sqrt{25}$

$(f\circ g^{-1})(3)=5$ ***A1*** ***N2***

Note: Award ***A0*** for multiple values, *eg* ± 5.

[3 marks]

Total [6 marks]

6. attempt to integrate which involves ln *(M1)*

eg $\ln(2x-5)$, $12\ln 2x-5$, $\ln 2x$

correct expression (accept absence of C)

eg $12\ln(2x-5)\frac{1}{2}+C$, $6\ln(2x-5)$ ***A2***

attempt to substitute $(4, 0)$ into **their** integrated f *(M1)*

eg $0=6\ln(2\times4-5)$, $0=6\ln(8-5)+C$

$C=-6\ln 3$ *(A1)*

$f(x)=6\ln(2x-5)-6\ln 3$ $\left(=6\ln\left(\frac{2x-5}{3}\right)\right)$ (accept $6\ln(2x-5)-\ln 3^6$) ***A1*** ***N5***

Note: Exception to the ***FT*** rule. Allow full ***FT*** on incorrect integration which must involve ln.

Total [6 marks]

7. (a) evidence of correct formula *(M1)*

eg $\log a-\log b=\log\frac{a}{b}$, $\log\left(\frac{40}{5}\right)$, $\log 8+\log 5-\log 5$

Note: Ignore missing or incorrect base.

correct working *(A1)*

eg $\log_2 8$, $2^3=8$

$\log_2 40-\log_2 5=3$ ***A1*** ***N2***

[3 marks]

(b) attempt to write 8 as a power of 2 (seen anywhere) *(M1)*

eg $(2^3)^{\log_2 5}$, $2^3=8$, 2^a

multiplying powers *(M1)*

eg $2^{3\log_2 5}$, $a\log_2 5$

correct working *(A1)*

eg $2^{\log_2 125}$, $\log_2 5^3$, $\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5}=125$ ***A1*** ***N3***

[4 marks]

Total [7 marks]

SECTION B

8. (a) (i) valid approach *(M1)*

eg $\begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, A − B, $\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AB} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ *A1* *N2*

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\vec{AB}$ *A2* *N2*

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $(x, y, z) = (1, -2, 3) + t(3, -1, 2)$, $\boldsymbol{r} = \begin{pmatrix} 1+6t \\ -2-2t \\ 3+4t \end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L_1 = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[4 marks]

(b) recognizing that scalar product = 0 (seen anywhere) ***R1***

correct calculation of scalar product *(A1)*
eg $6(3) - 2(-3) + 4p$, $18 + 6 + 4p$

correct working ***A1***
eg $24 + 4p = 0$, $4p = -24$

$p = -6$ ***AG*** ***N0***

[3 marks]

continued …

Question 8 continued

(c) setting lines equal **(M1)**

eg $L_1 = L_2$, $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$

any two correct equations with **different** parameters **A1A1**
eg $1+6t=-1+3s$, $-2-2t=2-3s$, $3+4t=15-6s$

attempt to solve **their** simultaneous equations **(M1)**

one correct parameter **A1**
eg $t=\frac{1}{2}$, $s=\frac{5}{3}$

attempt to substitute parameter into vector equation **(M1)**

eg $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $1+\frac{1}{2}\times 6$

$x = 4$ (accept $(4, -3, 5)$, ignore incorrect values for y and z) **A1** **N3**

[7 marks]

Total [14 marks]

9. (a) (i) attempt to find $\mathrm{P}(\text{red}) \times \mathrm{P}(\text{red})$ ***(M1)***

eg $\frac{3}{8} \times \frac{2}{7}$, $\frac{3}{8} \times \frac{3}{8}$, $\frac{3}{8} \times \frac{2}{8}$

$\mathrm{P}(\text{none green}) = \frac{6}{56} \left(= \frac{3}{28} \right)$ ***A1*** ***N2***

(ii) attempt to find $\mathrm{P}(\text{red}) \times \mathrm{P}(\text{green})$ ***(M1)***

eg $\frac{5}{8} \times \frac{3}{7}$, $\frac{3}{8} \times \frac{5}{8}$, $\frac{15}{56}$

recognizing two ways to get one red, one green ***(M1)***

eg $2\mathrm{P}(R) \times \mathrm{P}(G)$, $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$, $\frac{3}{8} \times \frac{5}{8} \times 2$

$\mathrm{P}(\text{exactly one green}) = \frac{30}{56} \left(= \frac{15}{28} \right)$ ***A1*** ***N2***

[5 marks]

(b) $\mathrm{P}(\text{both green}) = \frac{20}{56}$ (seen anywhere) ***(A1)***

correct substitution into formula for $\mathrm{E}(X)$ ***A1***

eg $0 \times \frac{6}{56} + 1 \times \frac{30}{56} + 2 \times \frac{20}{56}$, $\frac{30}{64} + \frac{50}{64}$

expected number of green marbles is $\frac{70}{56} \left(= \frac{5}{4} \right)$ ***A1*** ***N2***

[3 marks]

continued …

Question 9 continued

(c) (i) $P(\text{jar B}) = \frac{4}{6} \quad \left(= \frac{2}{3}\right)$ ***A1*** ***N1***

(ii) $P(\text{red} \mid \text{jar B}) = \frac{6}{8} \quad \left(= \frac{3}{4}\right)$ ***A1*** ***N1***

[2 marks]

(d) recognizing conditional probability ***(M1)***

eg $P(A|R)$, $\frac{P(\text{jar A and red})}{P(\text{red})}$, tree diagram

attempt to multiply along either branch (may be seen on diagram) ***(M1)***

eg $P(\text{jar A and red}) = \frac{1}{3} \times \frac{3}{8} \quad \left(= \frac{1}{8}\right)$

attempt to multiply along **other** branch ***(M1)***

eg $P(\text{jar B and red}) = \frac{2}{3} \times \frac{6}{8} \quad \left(= \frac{1}{2}\right)$

adding the probabilities of two mutually exclusive paths ***(A1)***

eg $P(\text{red}) = \frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8} \quad \left(= \frac{5}{8}\right)$

correct substitution

eg $P(\text{jar A}|\text{red}) = \dfrac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8}}, \ \dfrac{\frac{1}{8}}{\frac{5}{8}}$ ***A1***

$P(\text{jar A}|\text{red}) = \frac{1}{5}$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

10. (a) substitute 0 into f ***(M1)***
eg $\ln(0+1)$, $\ln 1$

$f(0)=0$ ***A1*** ***N2***
[2 marks]

(b) $f'(x)=\dfrac{1}{x^4+1}\times 4x^3$ (seen anywhere) ***A1A1***

> **Note:** Award ***A1*** for $\dfrac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x)>0$ (seen anywhere) ***R1***
eg $f'(x)>0$, diagram of signs

attempt to solve $f'(x)>0$ ***(M1)***
eg $4x^3=0$, $x^3>0$

f increasing for $x>0$ (accept $x\geq 0$) ***A1*** ***N1***
[5 marks]

(c) (i) substituting $x=1$ into f'' ***(A1)***
eg $\dfrac{4(3-1)}{(1+1)^2}$, $\dfrac{4\times 2}{4}$

$f''(1)=2$ ***A1*** ***N2***

(ii) valid interpretation of point of inflexion (seen anywhere) ***R1***
eg no change of sign in $f''(x)$, no change in concavity,
f' increasing both sides of zero

attempt to find $f''(x)$ for $x<0$ ***(M1)***
eg $f''(-1)$, $\dfrac{4(-1)^2\left(3-(-1)^4\right)}{\left((-1)^4+1\right)^2}$, diagram of signs

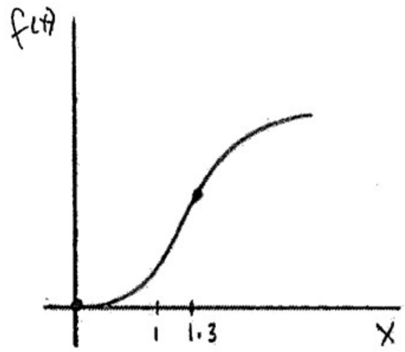
correct working leading to positive value ***A1***
eg $f''(-1)=2$, discussing signs of numerator **and** denominator

there is no point of inflexion at $x=0$ ***AG*** ***N0***
[5 marks]

continued …

Question 10 continued

(d)



A1A1A1 ***N3***

Notes: Award ***A1*** for shape concave up left of POI and concave down right of POI.
Only if this ***A1*** is awarded, then award the following:
A1 for curve through $(0, 0)$, ***A1*** for increasing throughout.
Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Total [15 marks]