AP CALCULUS
Test #6: Unit #6 – Basic Integration and Applications

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which *f*(*x*) is a real number.

Multiple Choice

1. If *k* is a constant, find its value if $\int_0^k \left(2kx - x^2\right)dx = 18$.

A. –9

B. –3

C. 3

D. 9

E. 18

2. After being poured into a cup, coffee cools so that its temperature, *T*(*t*), is represented by the function $T(t) = 70 + 110e^{-t/2}$, where *t* is measured in minutes and *T*(*t*) is measured in degrees Fahrenheit. What is the average temperature of the coffee during the first four minutes after being poured?

A. 470.226 °F

B. 1356.996 °F

C. 7.443 °F

D. 5427.984 °F

E. 117.557 °F

3. Which of the following integrals is/are true if $f(x)$ is a differentiable function on the open interval (a, b), c is on the open interval (a, b) , $\int_a^c f(x)dx = 6$ and $\int_c^b f(x)dx = -2$.

I. $\int_a^b f(x)dx = 4$ II. $\int_c^a \frac{1}{3} f(x)dx = \int_c^b f(x)dx$ III. $\int_b^a 2f(x)dx = 9$

A. I and II only

B. II only

C. I only

D. II and III only

E. III only

---

4. At time $t = 0$ water begins leaking from a tank at the rate of $L(t) = 5e^{-\frac{(t-3)^2}{2}}$ gallons per minute, where t is measured in minutes. How much water has leaked out of the tank after 5 minutes?

A. 0.621 gallons

B. 0.676 gallons

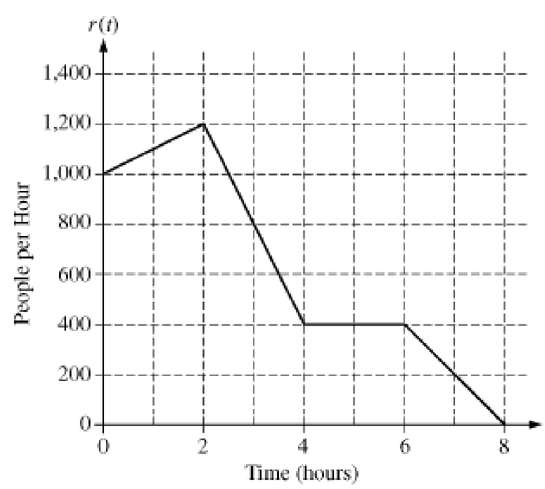
C. 1.353 gallons

D. 12.231 gallons

E. 15.769 gallons

---

5. The graph to the right represents the rate at which people arrive at an amusement park ride throughout the day, where t is measured in hours from the time the ride begins operation. If there were 275 people in line when the ride began operation, How many people have waited in line for the ride after 4 hours?



A. 3800
B. 675
C. 400
D. 4075
E. 600

6. Using a right Riemann sum over the given intervals, estimate $\int_5^{35} F(t)dt$.

t	5	13	22	27	35
$F(t)$	44	12	13	17	22

A. 730
B. 661
C. 564
D. 474
E. 325

7. At 10 a.m. the temperature at a ski resort begins to increase causing the snow to begin to melt at a rate defined by the equation $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. If there are 178 cubic yards of snow at that point, how much snow remains at 5 p.m. if no additional snow has been added and the temperature has continually increased throughout the day?

A. 90.646 cubic yards

B. 265.354 cubic yards

C. 96.177 cubic yards

D. 69.646 cubic yards

E. 73.128 cubic yards

Free Response

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}.$$

The rate at which people leave the same amusement part on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in number of hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At $t = 9$, there are no people in the park.

a. The price of admission to the park is \$15 until 5:00 p.m. After 5:00 p.m., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

b. Let $H(t) = \int_9^t (E(t) - L(t))dt$ for $9 \le t \le 23$. Find the values of $H(17)$ and $H'(17)$ and explain, using correct units, the meaning of both values in the context of the park.

c. During the hours that the park is open, $9 \le t \le 23$, what is the maximum number of people in the park at any given moment? Show your work and justify your answer.

AP CALCULUS
Test #6: Unit #6 – Basic Integration and Applications

A GRAPHING CALCULATOR IS **NOT** ALLOWED FOR THIS SECTION OF THE EXAM.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Multiple Choice

1. If $\int_0^3 f(x)dx = 6$ and $\int_3^5 f(x)dx = 4$, then $\int_0^5 (3 + 2f(x))dx =$

A. 10

B. 20

C. 23

D. 35

E. 50

---

2. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)dx =$

A. $-\frac{14}{3}$

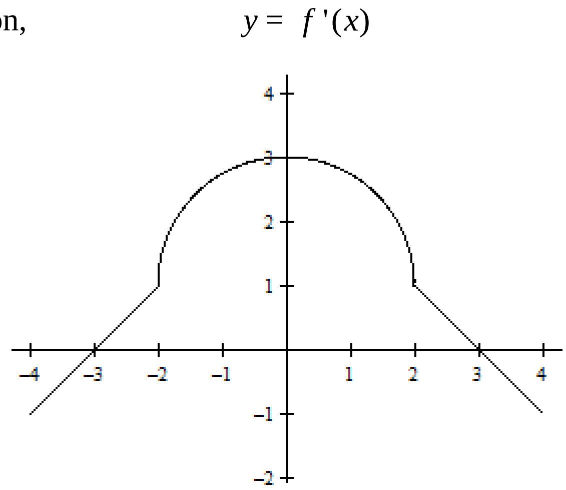
B. –2

C. 2

D. 4

E. $\frac{14}{3}$

3. Given to the right is the graph of the derivative of a function, $f'(x)$. If $f(0) = 9$, what is the value of $f(2)$?



A. $11 + 2\pi$

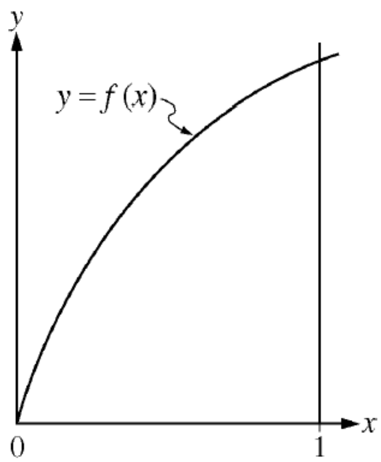
B. $-7 + 4\pi$

C. $-7 + \pi$

D. $3 + 4\pi$

E. $11 + \pi$

4. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)dx$, each using the same number of subintervals. The graph of the function f is shown in the figure to the right. Which of the sums give(s) an underestimate of the value of $\int_0^1 f(x)dx$?



I. Left Sum II. Right Sum III. Trapezoidal Sum

A. I only

B. II only

C. III only

D. I and III only

E. II and III only

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

5. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate of $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and a data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

   A. 64.9

   B. 68.2

   C. 114.9

   D. 116.6

   E. 118.2

---

x	0	2	4	6
$f(x)$	4	k	8	12

6. The function f is continuous on the closed interval [0, 6] and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$, found with 3 subintervals of equal length is 52. What is the value of k?

   A. 2

   B. 6

   C. 7

   D. 10

   E. 14

Use the table of values below to answer question 7. $f(x)$ is a twice-differentiable function for which values of f, f', and f'' are given.

x	–3	–2	–1	0	1
$f(x)$	7	3	1	3	7
$f'(x)$	–5	–3	0	3	5
$f''(x)$	2	–1	–3	–2	0

7. Find the value of $\int_{-3}^{1}\left[2f'(x)+3f''(x)\right]dx$.

A. 13

B. 30

C. 0

D. 9

E. 4

Free Response

A particle is moving along a straight path. The velocity of the particle for $0 \le t \le 30$ is shown in the table below for selected values of t and velocity is at a maximum at $t = 20$ sec. Answer the questions that follow.

t	0	3	6	9	12	15	18	21	24	27	30
$v(t)$ m/sec	0	7.5	10.1	12.5	13	13.5	14.1	14	13.9	13	12.2

a. Using the midpoints of five subintervals of equal length, approximate the value of $\frac{1}{30}\int_0^{30} v(t)dt$.

Using correct units, explain the meaning of the value of $\frac{1}{30}\int_0^{30} v(t)dt$.

b. Find the average acceleration of the particle over the interval $6 \le t \le 18$. Express your answer using correct units.

c. Find an approximation of $v'(6)$. Using correct units, explain what this value represents and state, providing justification, if the speed of the particle is increasing or decreasing at $t = 6$?

d. During what interval(s) of time is the acceleration negative? Justify your answer.