

## **Final Exam Review: Logarithms, Exponentials, Sequences, & Series**

---

Use this sheet in conjunction with your old notes, quizzes, and tests to review.

**Formulas/Identities:**

1. An arithmetic series has five terms. The first term is 2 and the last term is 32. Find the sum of the series.

**(Total 4 marks)**

2. Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \dots + 3750 = s.$$

Calculate  $s$ .

**(Total 6 marks)**

3. Find the sum of the infinite geometric series

$$\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \dots$$

**(Total 4 marks)**

4. Ashley and Billie are swimmers training for a competition.

- (a) Ashley trains for 12 hours in the first week. She decides to increase the amount of time she spends training by 2 hours each week. Find the total number of hours she spends training during the first 15 weeks.

**(3)**

- (b) Billie also trains for 12 hours in the first week. She decides to train for 10% longer each week than the previous week.

- (i) Show that in the third week she trains for 14.52 hours.

- (ii) Find the total number of hours she spends training during the first 15 weeks.

**(4)**

- (c) In which week will the time Billie spends training first exceed 50 hours?

**(4)**

**(Total 11 marks)**

5. Solve the equation  $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$ .

**(Total 4 marks)**

6. Given that  $\log_5 x = y$ , express each of the following in terms of  $y$ .

(a)  $\log_5 x^2$

(b)  $\log_5 \left(\frac{1}{x}\right)$

(c)  $\log_{25} x$

**(Total 6 marks)**

7. A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

**(Total 4 marks)**

8. The number of bacteria,  $n$ , in a dish, after  $t$  minutes is given by  $n = 800e^{0.13t}$ .

(a) Find the value of  $n$  when  $t = 0$ .

**(2)**

(b) Find the rate at which  $n$  is increasing when  $t = 15$ .

**(2)**

(c) After  $k$  minutes, the rate of increase in  $n$  is greater than 10 000 bacteria per minute. Find the least value of  $k$ , where  $k \in \mathbb{Z}$ .

**(4)**

**(Total 8 marks)**

9. Let  $\log_{10} P = x$ ,  $\log_{10} Q = y$  and  $\log_{10} R = z$ . Express  $\log_{10} \left(\frac{P}{QR^3}\right)^2$  in terms of  $x$ ,  $y$  and  $z$ .

**(Total 4 marks)**

10. Solve the equation  $\log_{27} x = 1 - \log_{27} (x - 0.4)$ .

**(Total 6 marks)**