Exponential & Logarithmic Functions Review

① $B = B_0(1+.023)^t$

To double, B would equal $2B_0$

$2B_0 = B_0(1.023)^t$

$2 = (1.023)^t$

$(1.023)^t = 2$

$\log_{1.023} 2 = t$

$t = 30.48$ or $t = 30$ (to the nearest minute)

② a. $V(5) = 10{,}000(.933)^5 = 7069.8 = 7070$ (3 sig figs)

b. $5000 = 10000(.933)^t$

$\frac{1}{2} = (.933)^t$

$(.933)^t = \frac{1}{2}$

$\log_{.933} \frac{1}{2} = t$

9.99 minutes $= t$ or 600 seconds

c. $500 = 10{,}000(.933)^t$

$.05 = (.933)^t$

$(.933)^t = .05$

$\log_{.933} .05 = t$

43.2 minutes $= t \approx$ 3/4 hour

d. i. $V(.001) = 10000(.933)^{.001} = 9999.306523\ldots$

$10000 - V(.001) = .693$ liters

ii. $\frac{.693 \text{ liters}}{.001 \text{ minutes}} = 693$ liters/minute

③ Omit, for now.

④ a. $p(0) = 100e^{.05(0)} = 100$ bacteria

b. omit - calculus question

⑤ a. $y = 1500(1+.0525)^t$. After 3 years, $t=3$

$y = 1500(1.0525)^3 = 1749$ (nearest franc)

b. $3000 = 1500(1.0525)^t$

$2 = (1.0525)^t$

$(1.0525)^t = 2$

$\log_{1.0525} 2 = t$

$t = 13.546 \rightarrow \underline{14}$ complete years

c. $3000 = 1500(1+r)^{10}$

$2 = (1+r)^{10}$

$\sqrt[10]{2} = \sqrt[10]{(1+r)^{10}}$

$\sqrt[10]{2} = 1+r$

$\sqrt[10]{2} - 1 = r$

or $2^{1/10} - 1 = r = .0718 = 7.18\%$

6. $e^{\ln 8x} = 2x^4$

7. $\log_w 11x - \log_w 2 = \log_w 11 + \log_w x - \log_w 2$

$\log_5 \sqrt[3]{m} + \log_5 \sqrt[7]{n} - \log_5 k^2$

$= \frac{1}{3}\log_5 m + \frac{1}{7}\log_5 n - 2\log_5 k$

9. $\frac{x-6}{6} = \frac{1}{2}$

$2x - 12 = 6$

$2x = 18$

$x = 9$

10. $(4^2)^{x+6} = (4^3)^{x-8}$

$4^{2x+12} = 4^{3x-24}$

$2x + 12 = 3x - 24$

$36 = x$

11. $e^x = \frac{29}{3}$

$x = \ln \frac{29}{3} \approx 2.27$

12. $(e^x)^2 + e^x - 6 = 0$

$(e^x + 3)(e^x - 2) = 0$

$e^x = -3$ not possible

$e^x = 2$

$x = \ln 2 \approx .693$

13. $3^{-1} = x - 1$

$\frac{1}{3} = \frac{x-1}{1}$

$3x - 3 = 1$

$3x = 4$

$x = \frac{4}{3}$

14. $\frac{1}{2}\ln(x+1) = 7$

$\ln(x+1) = 14$

$e^{14} = x + 1$

$e^{14} - 1 = x$

15. $\log_9 7x = 1$

$9 = 7x$

$\frac{9}{7} = x$

16. $\log_5 \frac{x+2}{x} = 2$

$25 = \frac{x+2}{x}$

$25x = x + 2$

$24x = 2$

$x = \frac{1}{12}$

17. $\ln(6x - 6) = 0$

$e^0 = 6x - 6$

$1 = 6x - 6$

$7 = 6x$

$\frac{7}{6} = x$

18. $\log_5 \frac{x+5}{x-3} = 1$

$\frac{5}{1} \quad \frac{x+5}{x-3}$

$5x-15 = x+5$

$4x = 20$

$x = 5$

19. $9^x - 10(3^x) + 9 = 0$

$(3^x)^2 - 10(3^x) + 9 = 0$

$w^2 - 10w + 9 = 0$

$(w-9)(w-1) = 0$

$(3^x - 9)(3^x - 1) = 0$

$3^x = 9 \qquad 3^x = 1$

$x = 2 \qquad x = 0$

20. $6x+8 = 6x+3$

$8 = 3$

No Solution!

21. $x^2 = 5x + 36$

$x^2 - 5x - 36 = 0$

$(x-9)(x+4) = 0$

$x = 9, -4$

22. $\log 3x = \log(4x-12)$

$3x = 4x - 12$

$12 = x$

23. $\log(x^2 - x) = \log 30$

$x^2 - x = 30$

$x^2 - x - 30 = 0$

$(x-6)(x+5) = 0$

$x = 6, -5$

24. $\ln(x^2+x) = \ln 6$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

25. $\log \frac{x+20}{2} = \log(3x+4)$

$\frac{x+20}{2} = 3x+4$

$x+20 = 6x+8$

$\frac{12}{5} = \frac{5x}{5}$

26. $\log \frac{x^2}{4} = \log 121$

$\frac{x^2}{4} = 121$

$x^2 = 484$

$x = \pm 22$

27. $\ln(x^2 - 5x - 6) = \ln(x-15)$

$x^2 - 5x - 6 = x - 15$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$

No Sol!

28. $\ln \frac{x-8}{x+7} = \ln \frac{x-10}{x+8}$

$\frac{x-8}{x+7} = \frac{x-10}{x+8}$

$x^2 - 3x - 70 = x^2 - 64$

$-3x = 6$

$x = -2$

No Sol!

29. $5400 = 2700\left(1+\frac{.07}{12}\right)^{12t}$

$2 = \left(1+\frac{.07}{12}\right)^{12t}$

$\log_{\left(1+\frac{.07}{12}\right)} 2 = 12t$

$9.9 \text{ yrs} = t$

30. $3200 = 2900\left(1+\frac{.09}{12}\right)^{12t}$

$\frac{32}{29} = \left(1+\frac{.09}{12}\right)^{12t}$

$\log_{\left(1+\frac{.09}{12}\right)} \frac{32}{29} = 12t$

$t = 1.1 \text{ years}$

31. $y = 2546000(1.013)^{7}$

$= 2{,}786{,}920$

32. $2 = 1(1.012)^{t}$

$\log_{1.012} 2 = t$

$58.1 \text{ years} = t$