Using the first and second derivatives

For each of the given functions, determine the interval(s) on which $f(x)$ is increasing and/or decreasing. Find all coordinates of the relative extrema. Locate any points of inflection and determine all intervals of concave up and/or down. **Provide justification for your answers.**

1) $f(x) = x^3 - 6x + 1$

$f'(x) = 3x^2 - 6$

$0 = 3(x^2 - 2)$

$x = \pm\sqrt{2}$

f': $+$ | $-\sqrt{2}$ | $-$ | $\sqrt{2}$ | $+$

inc: $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ since $f' > 0$.

dec: $(-\sqrt{2}, \sqrt{2})$ since $f' < 0$.

rel max at $x = -\sqrt{2}$ since f' changes from $+$ to $-$.

rel min at $x = \sqrt{2}$ since f' changes from $-$ to $+$.

$f''(x) = 6x$

$0 = 6x$

$x = 0$

f'': $-$ | 0 | $+$

f is concave up $(0, \infty)$ since $f'' > 0$.

f is concave down $(-\infty, 0)$ since $f'' < 0$.

Inflection point at $x = 0$ since f'' changes from ~~+~~ $-$ to $+$.

2) $g(x) = 3x^5 - 5x^3$

$g'(x) = 15x^4 - 15x^2$

$0 = 15x^2(x^2-1)$

$0 = 15x^2(x-1)(x+1)$

$x = 0, 1, -1$

g': $+$ | -1 | $-$ | 0 | $-$ | 1 | $+$

inc $(-\infty,-1) \cup (1,\infty)$ since $f'(x) > 0$.

dec $(-1,0) \cup (0,1)$ since $f'(x) < 0$.

rel max at $x = -1$ since $f'(x)$ changes from $+$ to $-$.

rel min at $x = 1$ since $f'(x)$ changes from $-$ to $+$.

$g''(x) = 60x^3 - 30x$

$0 = 30x(2x^2-1)$

$x = 0, \pm\sqrt{\frac{1}{2}}$

g'': $-$ | $-\sqrt{\frac{1}{2}}$ | $+$ | 0 | $\cancel{+}\ -$ | $\sqrt{\frac{1}{2}}$ | $+$

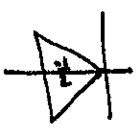
concave up $(-\sqrt{\frac{1}{2}}, 0) \cup (\sqrt{\frac{1}{2}}, \infty)$ since $f''(x) > 0$.

concave down $(-\infty, -\sqrt{\frac{1}{2}}) \cup (0, \sqrt{\frac{1}{2}})$ since $f''(x) < 0$.

Inflection pts at $x = \pm\sqrt{\frac{1}{2}}, 0$ since concavity changes.

★ 3) $f(\theta) = \theta + 2\sin\theta$, on $[0, 2\pi]$

$f'(\theta) = 1 + 2\cos\theta$



$0 = 1 + 2\cos\theta$

$\cos\theta = \frac{-1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

f': 0 | $+$ | $\frac{2\pi}{3}$ | $-$ | $\frac{4\pi}{3}$ | $+$ | 2π

inc $(0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$ since $f'(x) > 0$.

dec $(\frac{2\pi}{3}, \frac{4\pi}{3})$ since $f'(x) < 0$.

rel max at $x = \frac{2\pi}{3}$ since $f'(x)$ changes from $+$ to $-$.

rel min at $x = \frac{4\pi}{3}$ since $f'(x)$ changes from $-$ to $+$.

$f''(\theta) = -2\sin\theta$

$0 = -2\sin\theta$

$\theta = 0, \pi, 2\pi$

f'': 0 | $-$ | π | $+$ | 2π

concave up $(\pi, 2\pi)$ since $f''(x) > 0$.

concave down $(0, \pi)$ since $f''(x) < 0$.

Inflection pts at $x = \pi$ since concavity changes.