Exponential and Logarithmic Functions Review

1. A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

2. Initially a tank contains 10 000 litres of liquid. At the time $t = 0$ minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by

$$V = 10\,000\,(0.933^t).$$

(a) Find the value of V after 5 minutes.

(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank.

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen.

(d) (i) Find the value of $10\,000 - V$ when $t = 0.001$ minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid. Give your answer in litres per minute, correct to two significant figures.

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3. $1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed $3000.

4. The population p of bacteria at time t is given by $p = 100e^{0.05t}$.

Calculate

(a) the value of p when $t = 0$;

(b) the rate of increase of the population when $t = 10$.

5. Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc.

(b) How many complete years will it take for Michele's initial investment to double in value?

(c) What should the interest rate be if Michele's initial investment were to double in value in 10 years?

Evaluate without a calculator:

6) e^{ln8x^4}

Use properties of logarithms to expand the expression as much as possible. Where possible, evaluate without a calculator:

7) $\log_w\left(\frac{11x}{2}\right)$

8) $\log_5\left(\frac{\sqrt[3]{m}\sqrt[7]{n}}{k^2}\right)$

Solve the equations without a calculator:

9) $8^{\frac{x-6}{6}} = \sqrt{8}$

10) $16^{x+6} = 64^{x-8}$

Solve the equations. Consider how to leave your answer without a calculator.

11) $3e^x = 29$

12) $e^{2x} + e^x - 6 = 0$

13) $\log_3(x-1) = -1$

14) $\ln\sqrt{x+1} = 7$

15) $\log_9 7 + \log_9 x = 1$

16) $\log_5(x+2) - \log_5 x = 2$

17) $\ln 6 + \ln(x-1) = 0$

18) $\log_5(x+5) - \log_5(x-3) = 1$

19) $9^x - 10(3^x) = -9$

20) $\log_9(6x+8) = \log_9(6x+3)$

21) $\log_6 x^2 = \log_6(5x+36)$

22) $\log 3x = \log 4 + \log(x-3)$

23) $\log x + \log(x-1) = \log 30$

24) $\ln x + \ln(x+1) = \ln 6$

25) $\log(x+20) - \log 2 = \log(3x+4)$

26) $2\log x - \log 4 = \log 121$

27) $\ln(x-6) + \ln(x+1) = \ln(x-15)$

28) $\ln(x-8) - \ln(x+7) = \ln(x-10) - \ln(x+8)$

29) Find the time it takes a \$2700 investment to double if it is invested at 7% compounded monthly. Round to the nearest tenth of a year.

30) Find how long it takes a \$2900 investment to earn \$300 interest if it is invested at 9% compounded monthly. Round to the nearest tenth of a year.

31) The population of a particular country is growing at the rate of 1.3% per year. If 2 546 00 people lived there in 2007, how many people will there be in the year 2014? Give your answer to an appropriate degree of accuracy.

32) The population of a certain country is growing at a rate of 1.2% per year. How long will it take to double the population?

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