WRITING A MATHEMATICAL EXPLORATION

In addition to sitting examination papers, Mathematics SL students are also required to complete a **mathematical exploration**. This is a short report written by the student, based on a topic of his or her choice, and should focus on the mathematics of that topic. The mathematical exploration comprises 20% of the final mark.

The exploration should be approximately 6-12 pages long, and should be written at a level which is accessible to an audience of your peers. The exploration should also include a bibliography.

Group work should not be used for explorations. Each student's exploration is an individual piece of work.

When deciding on how to structure your exploration, you may wish to include the following sections:

Introduction: This section can be used to explain why the topic has been chosen, and to include any relevant background information.

Aim: A clear statement of intent should be given to provide perspective and direction to your exploration. This should be a short paragraph which outlines the problem or scenario under investigation.

Method and Results: This section can be used to describe the process which was followed to investigate the problem, as well as recording the unprocessed results of your investigations, in the form of a table, for example.

Analysis of Results: In this section, you should use graphs, diagrams, and calculations to analyse your results. Any graphs and diagrams should be included in the appropriate place in the report, and not attached as appendices at the end. You should also form some conjectures based on your analysis.

Conclusion: You should summarise your investigation, giving a clear response to your aim. You should also reflect on your exploration. Limitations and sources of error could be discussed, as well as potential for further exploration.

The exploration will be assessed against five assessment criteria. Refer to the Mathematics SL Subject Guide for more details.

The following two pages contain a short extract of a student's report, used with the permission of Wan Lin Oh. Please note that there is no single structure which must be followed to write a mathematical exploration. The extract displayed is only intended to illustrate some of the key features which should be included.

The electronic version of this extract contains further information, and can be accessed by clicking the icon alongside.

ELECTRONIC EXTRACT

This is an **extract** of a mathematics report used to demonstrate the components of a written report.

1. Title (and author)
A clear and concise description of the report

Population Trends in China
Written by Wan Lin Oh

2. Introduction (optional)
Include background information and definitions of key terms or variables used.

Aim

To determine the model that best fits the population of China from 1950 to 2008 by investigating different functions that best model the population of China from 1950 to 1995 (refer to *Table 1*) initially, and then re-evaluating and modifying this model to include additional data from 1983 to 2008.

Rationale

3. Aim and Rationale
Outline the purpose of the task in a clear and succinct manner.
Justify the exploration choice.

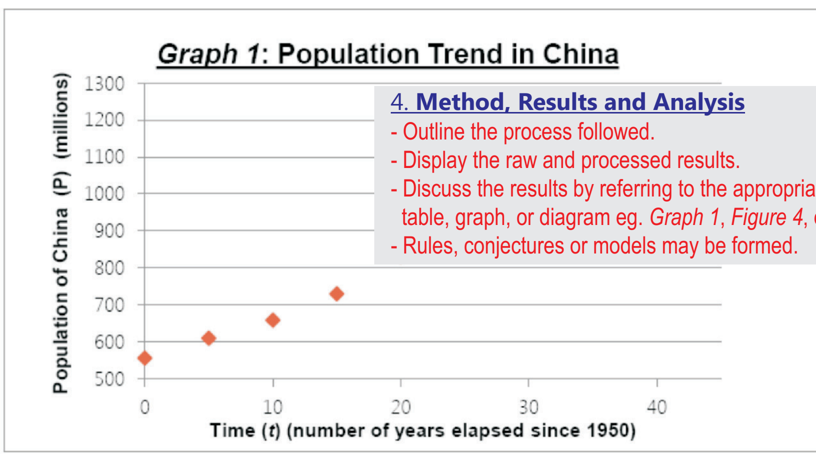
The history class had been discussi , cultural and social implications of China's "One Child Policy", introduce . This aroused the author's curiosity about the measurable impact that the policy may have had on China's population.

Table 1: Population of China from 1950 to 1995

Year (t)	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Population in millions (P)	554.8	609.0	657.5	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

Choosing a model

Values from *Table 1* were used to create *Graph 1*:



4. Method, Results and Analysis
- Outline the process followed.
- Display the raw and processed results.
- Discuss the results by referring to the appropriate table, graph, or diagram eg. *Graph 1*, *Figure 4*, etc.
- Rules, conjectures or models may be formed.

Graph 1 illustrates a positive correlation between the population of China and the number of years since 1950. This means that as time increases, the population of China also increases. *Graph 1* clearly shows that the model is not a linear function, because the graph has turning points and there is no fixed increase in corresponding to a fixed increase in P. Simple observation reveals that it is not a straight line. In addition, *Graph 1* illustrates that the function is not a power function ($P = at^b$) because it does not meet the requirements of a power function; for all positive b values, a power model must go through the origin, however *Graph 1* shows that the model's function does not pass through the origin of (0, 0).

There is a high possibility that the model could be a polynomial function because *Graph 1* indicates that there are turning point(s). A cubic and a quadratic function were then determined and compared.

Analytical Determination of Polynomial Model

As there is a high possibility that the model could be a cubic function (3rd degree polynomial function), an algebraic method can be used in order to determine the equation of the function. In order to determine this cubic equation, four points from the model will be used as there are four...

Conclusion

The aim of this investigation was to investigate a model that best fits the given data from 1950 to 2008. It was initially found that a 3rd degree polynomial function and an exponential function have a good possibility of fitting the given data from *Table 1* which is from year 1950 to 1995 by observing the data plots on the graph.

A cubic function (3rd degree polynomial function) was chosen eventually and consequently an algebraic method using simultaneous equations was developed to produce the equation of the function. Through this method, the equation of the cubic was deduced to be $P(t) = -0.007081t^3 + 0.5304t^2 + 5.263t + 554.8$. In addition, the use of technology was also included in this investigation to further enhance the development of the task by graphing the cubic function to determine how well the cubic function fitted the original data. The cubic graph was then compared with a quadratic function graph of $P(t) = 0.13t^2 + 8.95t + 554.8$. Ultimately, the cubic function was seen as the better fit compared to the quadratic model.

A researcher suggests that the population, P at time t can be modelled by $P(t) = \frac{K}{1+Le^{-Mt}}$. With the use of GeoGebra the parameters, K, L and M were found by trial and error to be 1590, 1.97 and 0.04 respectively. This consequently led to the equation of the logistic function of $P(t) = \frac{1590}{1+1.97e^{-0.04t}}$.

From the comparison of both the cubic and the logistic model, the cubic function was established to be a more accurate model for the given 1950 – 1995 data because the data points matched the model better, however the logistic model produced more likely values under extrapolation.

Additional data on population trends in China fro ... d by the International Monetary Fund (IMF) was given in Tabl ... graphed with the additional data points and compared. It v ... er model compared to the cubic model because it was able ... ulation of China much more precisely.

Subsequently a piecewise function was used becau ... have two distinctly different parts, each with a correspondir ... domain $0 < t \leq 30$. The researcher's model was modified to fit the data for $30 < t \leq 58$.

5. Conclusion and Limitations

- Summarise findings in response to the stated aim including restating any rules, conjectures, or models.
- Comment on any limitations to the approach used or of the findings.
- Considerations of extensions and connections to personal/previous knowledge may also contextualise the significance of the exploration.

The piecewise function was then defined as

$$P(t) \begin{cases} -0.007081t^3 + 0.5304t^2 + 5.263t + 554.8 & 0 < t \leq 30 \\ \dfrac{1590}{1+1.97e^{-0.04t}} & 30 < t \leq 58 \end{cases}$$

This modified model matched the data points of the population of China from 1950 to 2008 closely; the model also passed through both the minimum and the maximum of the given data. In addition, the modified model exhibited good long-term behaviour and was able to predict a sensible result beyond the known values.

Limitations

In this investigation, there were several limitations that should be taken into account. Firstly, the best fit model which is the piecewise function model does not take into account the possibility of natural disasters or diseases that may occur in China in the future which will lead to a mass decrease in population. Furthermore, the model also does not consider the population pressures in China such as the one child policy. The one child policy introduced in 1978 but applied in 1979 would cause a decrease in the population in the long term. It is shown in *Graph 14* that after 1979 (P_7), the rate at which the Chinese population is increasing is slower compared to the previous years. This is because this policy leads to an increase in the abortion rate due to many families' preference for males, as males are able to take over the family name. This will consequently lead to a gender imbalance, causing a decrease in population because of the increasing difficulty for Chinese males to find partners. In addition, the model of best fit does not consider the ... countries, allowing more Chinese people to live longer, which wil ... term.

6. References and acknowledgements

A list of sources of information either footnoted on the appropriate page or given in a bibliography at the end of the report.

[1]http://geography.about.com/od/populationgeography/a/onechild.htm