

**Example 19****Self Tutor**Find  $x$  exactly:

**a**  $e^x = 30$

**b**  $3e^{\frac{x}{2}} = 21$

**a**  $e^x = 30$   
 $\therefore x = \ln 30$

**b**  $3e^{\frac{x}{2}} = 21$   
 $\therefore e^{\frac{x}{2}} = 7$   
 $\therefore \frac{x}{2} = \ln 7$   
 $\therefore x = 2 \ln 7$

**EXERCISE 4E****1** Solve for  $x$ , giving an exact answer in base 10:

**a**  $2^x = 10$

**b**  $3^x = 20$

**c**  $4^x = 100$

**d**  $\left(\frac{1}{2}\right)^x = 0.0625$

**e**  $\left(\frac{3}{4}\right)^x = 0.1$

**f**  $10^x = 0.000\ 01$

**2** Solve for  $x$ , giving an exact answer:

**a**  $e^x = 10$

**b**  $e^x = 1000$

**c**  $2e^x = 0.3$

**d**  $e^{\frac{x}{2}} = 5$

**e**  $e^{2x} = 18$

**f**  $e^{-\frac{x}{2}} = 1$

**Example 20****Self Tutor**Consider the equation  $P = 200 \times 2^{0.04t}$ .**a** Rearrange the equation to give  $t$  in terms of  $P$ .**b** Hence find the value of  $t$  when  $P = 6$ .

$$\begin{aligned} \mathbf{a} \quad P &= 200 \times 2^{0.04t} \\ \therefore 2^{0.04t} &= \frac{P}{200} && \{\text{dividing both sides by } 200\} \\ \therefore \log 2^{0.04t} &= \log \left( \frac{P}{200} \right) && \{\text{finding the logarithm of each side}\} \\ \therefore 0.04t \times \log 2 &= \log \left( \frac{P}{200} \right) && \{\log(A)^n = n \log A\} \\ \therefore t &= \frac{\log \left( \frac{P}{200} \right)}{0.04 \times \log 2} \end{aligned}$$

$$\mathbf{b} \quad \text{When } P = 6, \quad t = \frac{\log \left( \frac{6}{200} \right)}{0.04 \times \log 2} \approx -126$$

**3** Consider the equation  $R = 200 \times 2^{0.25t}$ .**a** Rearrange the equation to give  $t$  in terms of  $R$ .**b** Hence find  $t$  when: **i**  $R = 600$  **ii**  $R = 1425$ **4** Consider the equation  $M = 20 \times 5^{-0.02x}$ .**a** Rearrange the equation to give  $x$  in terms of  $M$ .**b** Hence find  $x$  when: **i**  $M = 100$  **ii**  $M = 232$