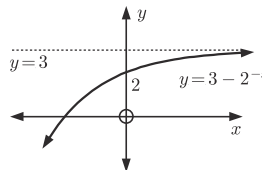


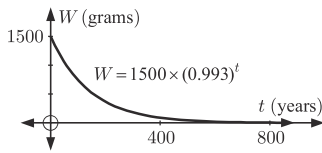
**b** Domain of  $f$ ,  $g$ , and  $h$  is  $\{x \mid x \in \mathbb{R}\}$   
 Range of  $f$  is  $\{y \mid y > 0\}$ , Range of  $g$  is  $\{y \mid y > 0\}$ ,  
 Range of  $h$  is  $\{y \mid y < 3\}$

**9 a**

$x$	-2	-1	0	1	2
$y$	-1	1	2	$2\frac{1}{2}$	$2\frac{3}{4}$

**b** as  $x \rightarrow \infty$ ,  $y \rightarrow 3$  (below); as  $x \rightarrow -\infty$ ,  $y \rightarrow -\infty$   
**c**  **d**  $y = 3$

**10 a** 1500 g  
**b i** 90.3 g  
**ii** 5.44 g  
**d** 386 years

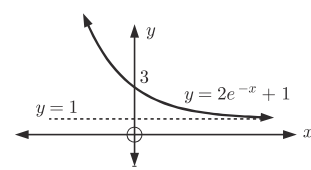


**REVIEW SET 3C**

- 1 a**  $x \approx 1.5$    **b**  $x \approx -0.6$    **c**  $x \approx 1.1$   
**2 a**  $a^{21}$    **b**  $p^4q^6$    **c**  $\frac{4b}{a^3}$   
**3 a**  $2^{-3}$    **b**  $2^7$    **c**  $2^{12}$    **4 a**  $\frac{1}{b^3}$    **b**  $\frac{1}{ab}$    **c**  $\frac{a}{b}$   
**5**  $2^{2x}$    **6 a**  $5^0$    **b**  $5^{\frac{3}{2}}$    **c**  $5^{-\frac{1}{4}}$    **d**  $5^{2a+6}$   
**7 a**  $1 + e^{2x}$    **b**  $2^{2x} + 10(2^x) + 25$    **c**  $x - 49$   
**8 a**  $x = 5$    **b**  $x = -4$    **9**  $k = \frac{3}{2}$   
**10 a**  $x = 4$    **b**  $x = -\frac{2}{5}$

**11 a**

$x$	-2	-1	0	1	2
$y$	15.8	6.44	3	1.74	1.27

**b** as  $x \rightarrow \infty$ ,  $y \rightarrow 1$  (above); as  $x \rightarrow -\infty$ ,  $y \rightarrow \infty$   
**c**  **d**  $y = 1$

**EXERCISE 4A**

- 1 a** 4   **b** -3   **c** 1   **d** 0   **e**  $\frac{1}{2}$    **f**  $\frac{1}{3}$   
**g**  $-\frac{1}{4}$    **h**  $1\frac{1}{2}$    **i**  $\frac{2}{3}$    **j**  $1\frac{1}{2}$    **k**  $1\frac{1}{3}$    **l**  $3\frac{1}{2}$   
**2 a**  $n$    **b**  $a + 2$    **c**  $1 - m$    **d**  $a - b$   
**3 a**  $10^{0.7782}$    **b**  $10^{1.7782}$    **c**  $10^{3.7782}$    **d**  $10^{-0.2218}$   
**e**  $10^{-2.2218}$    **f**  $10^{1.1761}$    **g**  $10^{3.1761}$    **h**  $10^{0.1761}$   
**i**  $10^{-0.8239}$    **j**  $10^{-3.8239}$   
**4 a i** 0.477   **ii** 2.477   **b**  $\log 300 = \log(3 \times 10^2)$   
**5 a i** 0.699   **ii** -1.301   **b**  $\log 0.05 = \log(5 \times 10^{-2})$   
**6 a**  $x = 100$    **b**  $x = 10$    **c**  $x = 1$   
**d**  $x = \frac{1}{10}$    **e**  $x = 10^{\frac{1}{2}}$    **f**  $x = 10^{-\frac{1}{2}}$   
**g**  $x = 10\,000$    **h**  $x = 0.000\,01$    **i**  $x \approx 6.84$   
**j**  $x \approx 140$    **k**  $x \approx 0.0419$    **l**  $x \approx 0.000\,631$

**EXERCISE 4B**

- 1 a**  $10^2 = 100$    **b**  $10^4 = 10\,000$    **c**  $10^{-1} = 0.1$   
**d**  $10^{\frac{1}{2}} = \sqrt{10}$    **e**  $2^3 = 8$    **f**  $3^2 = 9$   
**g**  $2^{-2} = \frac{1}{4}$    **h**  $3^{1.5} = \sqrt{27}$    **i**  $5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$   
**2 a**  $\log_2 4 = 2$    **b**  $\log_4 64 = 3$    **c**  $\log_5 25 = 2$   
**d**  $\log_7 49 = 2$    **e**  $\log_2 64 = 6$    **f**  $\log_2(\frac{1}{8}) = -3$   
**g**  $\log_{10} 0.01 = -2$    **h**  $\log_2(\frac{1}{2}) = -1$    **i**  $\log_3(\frac{1}{27}) = -3$   
**3 a** 5   **b** -2   **c**  $\frac{1}{2}$    **d** 3   **e** 6   **f** 7   **g** 2  
**h** 3   **i** -3   **j**  $\frac{1}{2}$    **k** 2   **l**  $\frac{1}{2}$    **m** 5   **n**  $\frac{1}{3}$   
**o**  $n, a > 0$    **p**  $\frac{1}{3}$    **q** -1,  $t > 0$    **r**  $\frac{3}{2}$    **s** 0  
**t** 1  
**4 a**  $\approx 2.18$    **b**  $\approx 1.40$    **c**  $\approx 1.87$    **d**  $\approx -0.0969$   
**5 a**  $x = 8$    **b**  $x = 2$    **c**  $x = 3$    **d**  $x = 14$   
**6 a** 2   **b** 2   **c** -1   **d**  $\frac{3}{4}$    **e**  $-\frac{1}{2}$    **f**  $\frac{5}{2}$   
**g**  $-\frac{3}{2}$    **h**  $-\frac{3}{4}$    **i** 2,  $x > 0$    **j**  $\frac{1}{2}$ ,  $x > 0$   
**k** 3,  $m > 0$    **l**  $\frac{3}{2}$ ,  $x > 0$    **m** -1,  $n > 0$   
**n** -2,  $a > 0$    **o**  $-\frac{1}{2}$ ,  $a > 0$    **p**  $\frac{5}{2}$ ,  $m > 0$

**EXERCISE 4C.1**

- 1 a**  $\log 16$    **b**  $\log 20$    **c**  $\log 8$    **d**  $\log \frac{p}{m}$   
**e** 1   **f**  $\log 2$    **g**  $\log 24$    **h**  $\log_2 6$   
**i**  $\log 0.4$    **j** 1   **k**  $\log 200$   
**l**  $\log(10^t \times w)$    **m**  $\log_m(\frac{40}{m^2})$    **n** 0  
**o**  $\log(0.005)$    **p**  $\log_5(\frac{5}{2})$    **q** 2   **r**  $\log 28$   
**2 a**  $\log 96$    **b**  $\log 72$    **c**  $\log 8$    **d**  $\log_3(\frac{25}{8})$   
**e** 1   **f**  $\log \frac{1}{2}$    **g**  $\log 20$    **h**  $\log 25$   
**i**  $\log_n(\frac{n^2}{10})$   
**3 a** 2   **b**  $\frac{3}{2}$    **c** 3   **d**  $\frac{1}{2}$    **e** -2   **f**  $-\frac{3}{2}$   
**4** For example, for **a**,  $\log 9 = \log 3^2 = 2 \log 3$   
**5 a**  $p + q$    **b**  $2q + r$    **c**  $2p + 3q$    **d**  $r + \frac{1}{2}q - p$   
**e**  $r - 5p$    **f**  $p - 2q$   
**6 a**  $x + z$    **b**  $z + 2y$    **c**  $x + z - y$    **d**  $2x + \frac{1}{2}y$   
**e**  $3y - \frac{1}{2}z$    **f**  $2z + \frac{1}{2}y - 3x$   
**7 a** 0.86   **b** 2.15   **c** 1.075

**EXERCISE 4C.2**

- 1 a**  $\log y = x \log 2$    **b**  $\log y \approx 1.30 + 3 \log b$   
**c**  $\log M = \log a + 4 \log d$    **d**  $\log T \approx 0.699 + \frac{1}{2} \log d$   
**e**  $\log R = \log b + \frac{1}{2} \log l$    **f**  $\log Q = \log a - n \log b$   
**g**  $\log y = \log a + x \log b$    **h**  $\log F \approx 1.30 - \frac{1}{2} \log n$   
**i**  $\log L = \log a + \log b - \log c$    **j**  $\log N = \frac{1}{2} \log a - \frac{1}{2} \log b$   
**k**  $\log S \approx 2.30 + t \log 2$    **l**  $\log y = m \log a - n \log b$   
**2 a**  $D = 2e$    **b**  $F = \frac{5}{t}$    **c**  $P = \sqrt{x}$    **d**  $M = b^2c$   
**e**  $B = \frac{m^3}{n^2}$    **f**  $N = \frac{1}{\sqrt[3]{p}}$    **g**  $P = 10x^3$    **h**  $Q = \frac{a^2}{x}$

- 3 a  $\log_2 y = \log_2 3 + x$       b  $x = \log_2 \left(\frac{y}{3}\right)$   
 c i  $x = 0$       ii  $x = 2$       iii  $x \approx 3.32$   
 4 a  $x = 9$       b  $x = 2$  or  $4$       c  $x = 25\sqrt{5}$   
 d  $x = 200$       e  $x = 5$       f  $x = 3$

**EXERCISE 4D.1**

- 1 a 2      b 3      c  $\frac{1}{2}$       d 0      e -1      f  $\frac{1}{3}$       g -2  
 h  $-\frac{1}{2}$   
 2 a 3      b 9      c  $\frac{1}{5}$       d  $\frac{1}{4}$   
 3  $x$  does not exist such that  $e^x = -2$  or  $0$   
 4 a  $a$       b  $a + 1$       c  $a + b$       d  $ab$       e  $a - b$   
 5 a  $e^{1.7918}$       b  $e^{4.0943}$       c  $e^{8.6995}$       d  $e^{-0.5108}$   
 e  $e^{-5.1160}$       f  $e^{2.7081}$       g  $e^{7.3132}$       h  $e^{0.4055}$   
 i  $e^{-1.8971}$       j  $e^{-8.8049}$   
 6 a  $x \approx 20.1$       b  $x \approx 2.72$       c  $x = 1$   
 d  $x \approx 0.368$       e  $x \approx 0.00674$       f  $x \approx 2.30$   
 g  $x \approx 8.54$       h  $x \approx 0.0370$

**EXERCISE 4D.2**

- 1 a  $\ln 45$       b  $\ln 5$       c  $\ln 4$       d  $\ln 24$   
 e  $\ln 1 = 0$       f  $\ln 30$       g  $\ln(4e)$       h  $\ln\left(\frac{6}{e}\right)$   
 i  $\ln 20$       j  $\ln(4e^2)$       k  $\ln\left(\frac{20}{e^2}\right)$       l  $\ln 1 = 0$   
 2 a  $\ln 972$       b  $\ln 200$       c  $\ln 1 = 0$       d  $\ln 16$       e  $\ln 6$   
 f  $\ln\left(\frac{1}{3}\right)$       g  $\ln\left(\frac{1}{2}\right)$       h  $\ln 2$       i  $\ln 16$   
 3 For example, for a,  $\ln 27 = \ln 3^3 = 3 \ln 3$   
 4 Hint:  $\ln d, \ln\left(\frac{e^2}{8}\right) = \ln e^2 - \ln 2^3$   
 5 a  $D = ex$       b  $F = \frac{e^2}{p}$       c  $P = \sqrt{x}$   
 d  $M = e^3 y^2$       e  $B = \frac{t^3}{e}$       f  $N = \frac{1}{\sqrt[3]{g}}$   
 g  $Q \approx 8.66x^3$       h  $D \approx 0.518n^{0.4}$

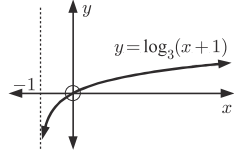
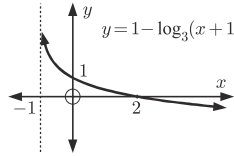
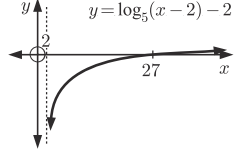
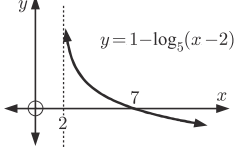
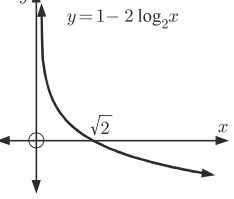
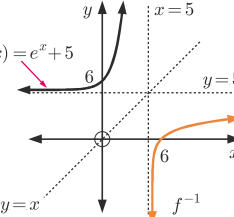
**EXERCISE 4E**

- 1 a  $x = \frac{1}{\log 2}$       b  $x = \frac{\log 20}{\log 3}$       c  $x = \frac{2}{\log 4}$   
 d  $x = 4$       e  $x = -\frac{1}{\log\left(\frac{3}{4}\right)}$       f  $x = -5$   
 2 a  $x = \ln 10$       b  $x = \ln 1000$       c  $x = \ln 0.15$   
 d  $x = 2 \ln 5$       e  $x = \frac{1}{2} \ln 18$       f  $x = 0$   
 3 a  $t = \frac{\log R - \log 200}{0.25 \log 2}$       b i  $t \approx 6.34$       ii  $t \approx 11.3$   
 4 a  $x = \frac{\log M - \log 20}{-0.02 \log 5}$       b i  $x = -50$       ii  $x \approx -76.1$   
 5 a  $x = -\frac{\log(0.03)}{\log 2}$       b  $x = \frac{10 \log\left(\frac{10}{3}\right)}{\log 5}$   
 c  $x = \frac{-4 \log\left(\frac{1}{8}\right)}{\log 3}$       d  $x = \frac{1}{2} \ln 42$   
 e  $x = -\frac{100}{3} \ln(0.001)$       f  $x = \frac{10}{3} \ln\left(\frac{27}{41}\right)$   
 6 a  $x = \ln 2$       b  $x = 0$       c  $x = \ln 2$  or  $\ln 3$       d  $x = 0$   
 e  $x = \ln 4$       f  $x = \ln\left(\frac{3+\sqrt{5}}{2}\right)$  or  $\ln\left(\frac{3-\sqrt{5}}{2}\right)$   
 7 a  $(\ln 3, 3)$       b  $(\ln 2, 5)$       c  $(0, 2)$  and  $(\ln 5, -2)$

**EXERCISE 4F**

- 1 a  $\approx 2.26$       b  $\approx -10.3$       c  $\approx -2.46$       d  $\approx 5.42$   
 2 a  $x \approx -4.29$       b  $x \approx 3.87$       c  $x \approx 0.139$   
 3 a  $x = \frac{\log 3}{\log 5}$       b  $x = \frac{\log\left(\frac{1}{8}\right)}{\log 3}$       c  $x = -1$       4  $x = 16$

**EXERCISE 4G**

- 1 a i domain is  $\{x \mid x > -1\}$ , range is  $\{y \mid y \in \mathbb{R}\}$       iii   
 ii VA is  $x = -1$ ,  $x$  and  $y$ -intercepts 0  
 iv  $x = -\frac{2}{3}$   
 v  $f^{-1}(x) = 3^x - 1$   
 b i domain is  $\{x \mid x > -1\}$ , range is  $\{y \mid y \in \mathbb{R}\}$       iii   
 ii VA is  $x = -1$ ,  $x$ -intercept 2,  $y$ -intercept 1  
 iv  $x = 8$   
 v  $f^{-1}(x) = 3^{1-x} - 1$   
 c i domain is  $\{x \mid x > 2\}$ , range is  $\{y \mid y \in \mathbb{R}\}$       iii   
 ii VA is  $x = 2$ ,  $x$ -intercept 27, no  $y$ -intercept  
 iv  $x = 7$   
 v  $f^{-1}(x) = 5^{2+x} + 2$   
 d i domain is  $\{x \mid x > 2\}$ , range is  $\{y \mid y \in \mathbb{R}\}$       iii   
 ii VA is  $x = 2$ ,  $x$ -intercept 7, no  $y$ -intercept  
 iv  $x = 27$   
 v  $f^{-1}(x) = 5^{1-x} + 2$   
 e i domain is  $\{x \mid x > 0\}$ , range is  $\{y \mid y \in \mathbb{R}\}$       iii   
 ii VA is  $x = 0$ ,  $x$ -intercepts  $\pm\sqrt{2}$ , no  $y$ -intercept  
 iv  $x = \pm 2$   
 v  $f^{-1}(x) = 2^{\frac{1-x}{2}}$   
 2 a i  $f^{-1}(x) = \ln(x - 5)$       ii   
 iii domain of  $f$  is  $\{x \mid x \in \mathbb{R}\}$ , range is  $\{y \mid y > 5\}$   
 domain of  $f^{-1}$  is  $\{x \mid x > 5\}$ , range is  $\{y \mid y \in \mathbb{R}\}$