- $c^{n\log_c A}$
  $= (c^{\log_c A})^n$
  $= A^n$
  $= c^{\log_c(A^n)}$
  $\therefore \quad n\log_c A = \log_c(A^n)$

Example 7 Self Tutor

Use the laws of logarithms to write the following as a single logarithm or as an integer:

a $\log 5 + \log 3$ **b** $\log_3 24 - \log_3 8$ **c** $\log_2 5 - 1$

a
$\log 5 + \log 3$
$= \log(5 \times 3)$
$= \log 15$

b
$\log_3 24 - \log_3 8$
$= \log_3\left(\frac{24}{8}\right)$
$= \log_3 3$
$= 1$

c
$\log_2 5 - 1$
$= \log_2 5 - \log_2 2^1$
$= \log_2\left(\frac{5}{2}\right)$

Example 8 Self Tutor

Simplify by writing as a single logarithm or as a rational number:

a $2\log 7 - 3\log 2$ **b** $2\log 3 + 3$ **c** $\dfrac{\log 8}{\log 4}$

a
$2\log 7 - 3\log 2$
$= \log(7^2) - \log(2^3)$
$= \log 49 - \log 8$
$= \log\left(\frac{49}{8}\right)$

b
$2\log 3 + 3$
$= \log(3^2) + \log(10^3)$
$= \log 9 + \log 1000$
$= \log(9000)$

c
$\dfrac{\log 8}{\log 4} = \dfrac{\log 2^3}{\log 2^2}$
$= \dfrac{3\log 2}{2\log 2}$
$= \frac{3}{2}$

EXERCISE 4C.1

1 Write as a single logarithm or as an integer:

a $\log 8 + \log 2$ **b** $\log 4 + \log 5$ **c** $\log 40 - \log 5$
d $\log p - \log m$ **e** $\log_4 8 - \log_4 2$ **f** $\log 5 + \log(0.4)$
g $\log 2 + \log 3 + \log 4$ **h** $1 + \log_2 3$ **i** $\log 4 - 1$
j $\log 5 + \log 4 - \log 2$ **k** $2 + \log 2$ **l** $t + \log w$
m $\log_m 40 - 2$ **n** $\log_3 6 - \log_3 2 - \log_3 3$ **o** $\log 50 - 4$
p $3 - \log_5 50$ **q** $\log_5 100 - \log_5 4$ **r** $\log\left(\frac{4}{3}\right) + \log 3 + \log 7$

2 Write as a single logarithm or integer:

a $5\log 2 + \log 3$ **b** $2\log 3 + 3\log 2$ **c** $3\log 4 - \log 8$
d $2\log_3 5 - 3\log_3 2$ **e** $\frac{1}{2}\log_6 4 + \log_6 3$ **f** $\frac{1}{3}\log\left(\frac{1}{8}\right)$
g $3 - \log 2 - 2\log 5$ **h** $1 - 3\log 2 + \log 20$ **i** $2 - \frac{1}{2}\log_n 4 - \log_n 5$

3 Simplify without using a calculator:

a $\dfrac{\log 4}{\log 2}$ **b** $\dfrac{\log_5 27}{\log_5 9}$ **c** $\dfrac{\log 8}{\log 2}$ **d** $\dfrac{\log 3}{\log 9}$ **e** $\dfrac{\log_3 25}{\log_3(0.2)}$ **f** $\dfrac{\log_4 8}{\log_4(0.25)}$

Check your answers using a calculator.

Example 9 **Self Tutor**

Show that:

a $\log\left(\frac{1}{9}\right) = -2\log 3$ **b** $\log 500 = 3 - \log 2$

a
$$\begin{aligned}&\log\left(\tfrac{1}{9}\right)\\ &= \log(3^{-2})\\ &= -2\log 3\end{aligned}$$

b
$$\begin{aligned}&\log 500\\ &= \log\left(\tfrac{1000}{2}\right)\\ &= \log 1000 - \log 2\\ &= \log 10^3 - \log 2\\ &= 3 - \log 2\end{aligned}$$

4 Show that:

a $\log 9 = 2\log 3$ **b** $\log\sqrt{2} = \frac{1}{2}\log 2$ **c** $\log\left(\frac{1}{8}\right) = -3\log 2$

d $\log\left(\frac{1}{5}\right) = -\log 5$ **e** $\log 5 = 1 - \log 2$ **f** $\log 5000 = 4 - \log 2$

5 If $p = \log_b 2$, $q = \log_b 3$, and $r = \log_b 5$ write in terms of p, q, and r:

a $\log_b 6$ **b** $\log_b 45$ **c** $\log_b 108$

d $\log_b\left(\frac{5\sqrt{3}}{2}\right)$ **e** $\log_b\left(\frac{5}{32}\right)$ **f** $\log_b(0.\overline{2})$

6 If $\log_2 P = x$, $\log_2 Q = y$, and $\log_2 R = z$ write in terms of x, y, and z:

a $\log_2(PR)$ **b** $\log_2(RQ^2)$ **c** $\log_2\left(\dfrac{PR}{Q}\right)$

d $\log_2(P^2\sqrt{Q})$ **e** $\log_2\left(\dfrac{Q^3}{\sqrt{R}}\right)$ **f** $\log_2\left(\dfrac{R^2\sqrt{Q}}{P^3}\right)$

7 If $\log_t M = 1.29$ and $\log_t N^2 = 1.72$ find:

a $\log_t N$ **b** $\log_t(MN)$ **c** $\log_t\left(\dfrac{N^2}{\sqrt{M}}\right)$

LOGARITHMIC EQUATIONS

We can use the laws of logarithms to write equations in a different form. This can be particularly useful if an unknown appears as an exponent.

For the logarithmic function, for every value of y, there is only one corresponding value of x. We can therefore take the logarithm of both sides of an equation without changing the solution. However, we can only do this if both sides are positive.