EXERCISE 3G.1

GRAPHING PACKAGE

1 The weight W_t of bacteria in a culture t hours after establishment is given by $W_t = 100 \times 2^{0.1t}$ grams.

- **a** Find the initial weight.
- **b** Find the weight after: **i** 4 hours **ii** 10 hours **iii** 24 hours.
- **c** Sketch the graph of W_t against t using the results of **a** and **b** only.
- **d** Use technology to graph $\text{Y}_1 = 100 \times 2^{0.1\text{X}}$ and check your answers to **a**, **b**, and **c**.

2 A breeding program to ensure the survival of pygmy possums is established with an initial population of 50 (25 pairs). From a previous program, the expected population P_n in n years' time is given by $P_n = P_0 \times 2^{0.3n}$.

- **a** What is the value of P_0?
- **b** What is the expected population after: **i** 2 years **ii** 5 years **iii** 10 years?
- **c** Sketch the graph of P_n against n using **a** and **b** only.
- **d** Use technology to graph $\text{Y}_1 = 50 \times 2^{0.3\text{X}}$ and check your answers to **b**.

3 A species of bear is introduced to a large island off Alaska where previously there were no bears. 6 pairs of bears were introduced in 1998. It is expected that the population will increase according to $B_t = B_0 \times 2^{0.18t}$ where t is the time since the introduction.

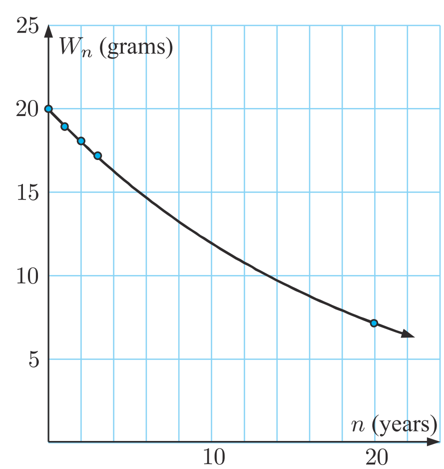
- **a** Find B_0.
- **b** Find the expected bear population in 2018.
- **c** Find the expected percentage increase from 2008 to 2018.

4 The speed V_t of a chemical reaction is given by $V_t = V_0 \times 2^{0.05t}$ where t is the temperature in °C.

- **a** Find the reaction speed at: **i** 0°C **ii** 20°C.
- **b** Find the percentage increase in reaction speed at 20°C compared with 0°C.
- **c** Find $\left(\dfrac{V_{50} - V_{20}}{V_{20}}\right) \times 100\%$ and explain what this calculation means.

DECAY

Consider a radioactive substance with original weight 20 grams. It *decays* or reduces by 5% each year. The multiplier for this is 95% or 0.95.



If W_n is the weight after n years, then:

$$\begin{aligned}
W_0 &= 20 \text{ grams} \\
W_1 &= W_0 \times 0.95 = 20 \times 0.95 \text{ grams} \\
W_2 &= W_1 \times 0.95 = 20 \times (0.95)^2 \text{ grams} \\
W_3 &= W_2 \times 0.95 = 20 \times (0.95)^3 \text{ grams} \\
&\vdots \\
W_{20} &= 20 \times (0.95)^{20} \approx 7.2 \text{ grams} \\
&\vdots \\
W_{100} &= 20 \times (0.95)^{100} \approx 0.1 \text{ grams}
\end{aligned}$$

and from this pattern we see that $W_n = 20 \times (0.95)^n$.

EXERCISE 3G.2

1 The weight of a radioactive substance t years after being set aside is given by $W(t) = 250 \times (0.998)^t$ grams.

a How much radioactive substance was initially set aside?

b Determine the weight of the substance after:

i 400 years **ii** 800 years **iii** 1200 years.

c Sketch the graph of $W(t)$ for $t \geqslant 0$ using **a** and **b** only.

d Use your graph or graphics calculator to find how long it takes for the substance to decay to 125 grams.

2 The temperature T of a liquid which has been placed in a refrigerator is given by $T(t) = 100 \times 2^{-0.02t}$ °C where t is the time in minutes.

a Find the initial temperature of the liquid.

b Find the temperature after:

i 15 minutes **ii** 20 minutes **iii** 78 minutes.

c Sketch the graph of $T(t)$ for $t \geqslant 0$ using **a** and **b** only.

3 Answer the **Opening Problem** on page **82**.

4 The weight W_t grams of radioactive substance remaining after t years is given by $W_t = 1000 \times 2^{-0.03t}$ grams.

a Find the initial weight of the radioactive substance.

b Find the weight remaining after:

i 10 years **ii** 100 years **iii** 1000 years.

c Graph W_t against t using **a** and **b** only.

d Use your graph or graphics calculator to find the time when 10 grams of the substance remains.

e Write an expression for the amount of substance that has decayed after t years.

5 The weight W_t of a radioactive uranium-235 sample remaining after t years is given by the formula $W_t = W_0 \times 2^{-0.0002t}$ grams, $t \geqslant 0$. Find:

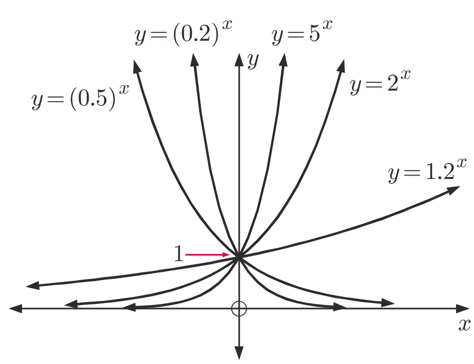
a the original weight **b** the percentage weight loss after 1000 years

c the time required until $\frac{1}{512}$ of the sample remains.

H THE NATURAL EXPONENTIAL e^x

We have seen that the simplest exponential functions are of the form $f(x) = b^x$ where $b > 0$, $b \neq 1$.

Graphs of some of these functions are shown alongside.



We can see that for all positive values of the base b, the graph is always positive.

Hence $b^x > 0$ for all $b > 0$.

There are an infinite number of possible choices for the base number.