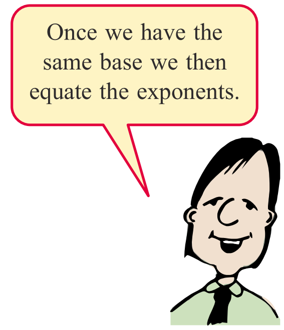
Example 17 Self Tutor

Solve for x:

a $2^x = 16$ **b** $3^{x+2} = \frac{1}{27}$

a
$$2^x = 16$$
$$\therefore \quad 2^x = 2^4$$
$$\therefore \quad x = 4$$

b
$$3^{x+2} = \frac{1}{27}$$
$$\therefore \quad 3^{x+2} = 3^{-3}$$
$$\therefore \quad x+2 = -3$$
$$\therefore \quad x = -5$$



Example 18 Self Tutor

Solve for x:

a $4^x = 8$ **b** $9^{x-2} = \frac{1}{3}$

a
$$4^x = 8$$
$$\therefore \quad (2^2)^x = 2^3$$
$$\therefore \quad 2^{2x} = 2^3$$
$$\therefore \quad 2x = 3$$
$$\therefore \quad x = \frac{3}{2}$$

b
$$9^{x-2} = \frac{1}{3}$$
$$\therefore \quad (3^2)^{x-2} = 3^{-1}$$
$$\therefore \quad 3^{2(x-2)} = 3^{-1}$$
$$\therefore \quad 2(x-2) = -1$$
$$\therefore \quad 2x-4 = -1$$
$$\therefore \quad 2x = 3$$
$$\therefore \quad x = \frac{3}{2}$$

EXERCISE 3E

1 Solve for x:

a $2^x = 8$ **b** $5^x = 25$ **c** $3^x = 81$ **d** $7^x = 1$

e $3^x = \frac{1}{3}$ **f** $2^x = \sqrt{2}$ **g** $5^x = \frac{1}{125}$ **h** $4^{x+1} = 64$

i $2^{x-2} = \frac{1}{32}$ **j** $3^{x+1} = \frac{1}{27}$ **k** $7^{x+1} = 343$ **l** $5^{1-2x} = \frac{1}{5}$

2 Solve for x:

a $8^x = 32$ **b** $4^x = \frac{1}{8}$ **c** $9^x = \frac{1}{27}$ **d** $25^x = \frac{1}{5}$

e $27^x = \frac{1}{9}$ **f** $16^x = \frac{1}{32}$ **g** $4^{x+2} = 128$ **h** $25^{1-x} = \frac{1}{125}$

i $4^{4x-1} = \frac{1}{2}$ **j** $9^{x-3} = 27$ **k** $(\frac{1}{2})^{x+1} = 8$ **l** $(\frac{1}{3})^{x+2} = 9$

m $81^x = 27^{-x}$ **n** $(\frac{1}{4})^{1-x} = 32$ **o** $(\frac{1}{7})^x = 49$ **p** $(\frac{1}{3})^{x+1} = 243$

3 Solve for x, if possible:

a $4^{2x+1} = 8^{1-x}$ **b** $9^{2-x} = (\frac{1}{3})^{2x+1}$ **c** $2^x \times 8^{1-x} = \frac{1}{4}$

4 Solve for x:

a $3 \times 2^x = 24$ **b** $7 \times 2^x = 56$ **c** $3 \times 2^{x+1} = 24$

d $12 \times 3^{-x} = \frac{4}{3}$ **e** $4 \times (\frac{1}{3})^x = 36$ **f** $5 \times (\frac{1}{2})^x = 20$

Example 19 Self Tutor

Solve for x: $4^x + 2^x - 20 = 0$

$$
\begin{aligned}
& 4^x + 2^x - 20 = 0 \\
\therefore\ & (2^x)^2 + 2^x - 20 = 0 && \{\text{compare } a^2 + a - 20 = 0\} \\
\therefore\ & (2^x - 4)(2^x + 5) = 0 && \{a^2 + a - 20 = (a-4)(a+5)\} \\
\therefore\ & 2^x = 4 \text{ or } 2^x = -5 \\
\therefore\ & 2^x = 2^2 && \{2^x \text{ cannot be negative}\} \\
\therefore\ & x = 2
\end{aligned}
$$

5 Solve for x:

a $4^x - 6(2^x) + 8 = 0$ **b** $4^x - 2^x - 2 = 0$ **c** $9^x - 12(3^x) + 27 = 0$

d $9^x = 3^x + 6$ **e** $25^x - 23(5^x) - 50 = 0$ **f** $49^x + 1 = 2(7^x)$

Check your answers using technology. You can get instructions for doing this by clicking on the icon.

GRAPHICS CALCULATOR INSTRUCTIONS

F EXPONENTIAL FUNCTIONS

We have already seen how to evaluate b^n when $n \in \mathbb{Q}$, or in other words when n is a rational number.

But what about b^n when $n \in \mathbb{R}$, so n is real but not necessarily rational?

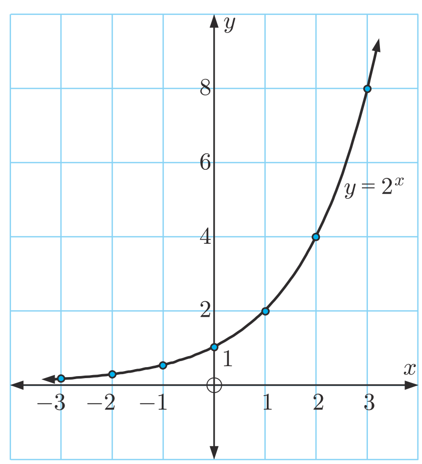
To answer this question, we can look at graphs of exponential functions.

The most simple general **exponential function** has the form $y = b^x$ where $b > 0$, $b \neq 1$.

For example, $y = 2^x$ is an exponential function.

We construct a table of values from which we graph the function:

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



When $x = -10$, $y = 2^{-10} \approx 0.001$.
When $x = -50$, $y = 2^{-50} \approx 8.88 \times 10^{-16}$.

As x becomes large and negative, the graph of $y = 2^x$ approaches the x-axis from above but never touches it, since 2^x becomes very small but never zero.

So, as $x \to -\infty$, $y \to 0^+$.

We say that $y = 2^x$ is '**asymptotic** to the x-axis' or '$y = 0$ is a **horizontal asymptote**'.

We now have a well-defined meaning for b^n where $b, n \in \mathbb{R}$ because simple exponential functions have smooth increasing or decreasing graphs.