FACTORISATION AND SIMPLIFICATION

Example 13 Self Tutor

Factorise: **a** $2^{n+3}+2^n$ **b** $2^{n+3}+8$ **c** $2^{3n}+2^{2n}$

a $2^{n+3}+2^n$
$=2^n 2^3+2^n$
$=2^n(2^3+1)$
$=2^n\times 9$

b $2^{n+3}+8$
$=2^n 2^3+8$
$=8(2^n)+8$
$=8(2^n+1)$

c $2^{3n}+2^{2n}$
$=2^{2n}2^n+2^{2n}$
$=2^{2n}(2^n+1)$

Example 14 Self Tutor

Factorise: **a** 4^x-9 **b** $9^x+4(3^x)+4$

a 4^x-9
$=(2^x)^2-3^2$ {compare $a^2-b^2=(a+b)(a-b)$}
$=(2^x+3)(2^x-3)$

b $9^x+4(3^x)+4$
$=(3^x)^2+4(3^x)+4$ {compare a^2+4a+4}
$=(3^x+2)^2$ {as $a^2+4a+4=(a+2)^2$}

EXERCISE 3D.2

1 Factorise:

a $5^{2x}+5^x$ **b** $3^{n+2}+3^n$ **c** 7^n+7^{3n}
d $5^{n+1}-5$ **e** $6^{n+2}-6$ **f** $4^{n+2}-16$

2 Factorise:

a 9^x-4 **b** 4^x-25 **c** $16-9^x$
d $25-4^x$ **e** 9^x-4^x **f** $4^x+6(2^x)+9$
g $9^x+10(3^x)+25$ **h** $4^x-14(2^x)+49$ **i** $25^x-4(5^x)+4$

3 Factorise:

a $4^x+9(2^x)+18$ **b** 4^x-2^x-20 **c** $9^x+9(3^x)+14$
d $9^x+4(3^x)-5$ **e** 25^x+5^x-2 **f** $49^x-7^{x+1}+12$

Example 15 Self Tutor

Simplify: **a** $\frac{6^n}{3^n}$ **b** $\frac{4^n}{6^n}$

a $\frac{6^n}{3^n}$ $=\frac{2^n\cancel{3^n}}{\cancel{3^n}_1}$ $=2^n$ **or** $\frac{6^n}{3^n}$ $=\left(\frac{6}{3}\right)^n$ $=2^n$

b $\frac{4^n}{6^n}$ $=\frac{\cancel{2^n}2^n}{\cancel{2^n}3^n}$ $=\frac{2^n}{3^n}$ **or** $\frac{4^n}{6^n}$ $=\left(\frac{4}{6}\right)^n$ $=\left(\frac{2}{3}\right)^n$

Example 16 **Self Tutor**

Simplify: **a** $\dfrac{3^n+6^n}{3^n}$ **b** $\dfrac{2^{m+2}-2^m}{2^m}$ **c** $\dfrac{2^{m+3}+2^m}{9}$

a
$$\frac{3^n+6^n}{3^n} = \frac{3^n+2^n3^n}{3^n} = \frac{\cancel{3^n}(1+2^n)}{\cancel{3^n}_1} = 1+2^n$$

b
$$\frac{2^{m+2}-2^m}{2^m} = \frac{2^m2^2-2^m}{2^m} = \frac{\cancel{2^m}(4-1)}{\cancel{2^m}_1} = 3$$

c
$$\frac{2^{m+3}+2^m}{9} = \frac{2^m2^3+2^m}{9} = \frac{2^m\cancel{(8+1)}}{\cancel{9}_1} = 2^m$$

4 Simplify:

a $\dfrac{12^n}{6^n}$ **b** $\dfrac{20^a}{2^a}$ **c** $\dfrac{6^b}{2^b}$ **d** $\dfrac{4^n}{20^n}$

e $\dfrac{35^x}{7^x}$ **f** $\dfrac{6^a}{8^a}$ **g** $\dfrac{5^{n+1}}{5^n}$ **h** $\dfrac{5^{n+1}}{5}$

5 Simplify:

a $\dfrac{6^m+2^m}{2^m}$ **b** $\dfrac{2^n+12^n}{2^n}$ **c** $\dfrac{8^n+4^n}{2^n}$

d $\dfrac{12^x-3^x}{3^x}$ **e** $\dfrac{6^n+12^n}{1+2^n}$ **f** $\dfrac{5^{n+1}-5^n}{4}$

g $\dfrac{5^{n+1}-5^n}{5^n}$ **h** $\dfrac{4^n-2^n}{2^n}$ **i** $\dfrac{2^n-2^{n-1}}{2^n}$

6 Simplify:

a $2^n(n+1)+2^n(n-1)$ **b** $3^n\left(\dfrac{n-1}{6}\right)-3^n\left(\dfrac{n+1}{6}\right)$

E EXPONENTIAL EQUATIONS

An **exponential equation** is an equation in which the unknown occurs as part of the index or exponent.

For example: $2^x=8$ and $30\times3^x=7$ are both exponential equations.

There are a number of methods we can use to solve exponential equations. These include graphing, using technology, and by using **logarithms**, which we will study in **Chapter 4**. However, in some cases we can solve algebraically by the following observation:

If $2^x=8$ then $2^x=2^3$. Thus $x=3$, and this is the only solution.

If the base numbers are the same, we can **equate exponents**.
If $\boldsymbol{a^x=a^k}$ then $\boldsymbol{x=k}$.