Exercise 11C.2

#3 a. $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 =$

$$\cos^2\theta + \cancel{2\sin\theta\cos\theta} + \sin^2\theta + \cos^2\theta - \cancel{2\sin\theta\cos\theta} + \sin^2\theta$$

$$\underbrace{\cos^2\theta + \sin^2\theta}_{1} + \underbrace{\cos^2\theta + \sin^2\theta}_{1} = 2$$

b. $(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2 =$

$$4\sin^2\theta + \cancel{12\sin\theta\cos\theta} + 9\cos^2\theta + 9\sin^2\theta - \cancel{12\sin\theta\cos\theta} + 4\cos^2\theta$$

$$4\sin^2\theta + 4\cos^2\theta + 9\cos^2\theta + 9\sin^2\theta$$

$$4(\underline{\sin^2\theta + \cos^2\theta}) + 9(\underline{\cos^2\theta + \sin^2\theta})$$

$$4(1) + 9(1) = 13$$

c. $(1 - \cos\theta)\left(1 + \dfrac{1}{\cos\theta}\right) =$

$$1 + \frac{1}{\cos\theta} - \cos\theta - \frac{\cos\theta}{\cos\theta} =$$

$$\cancel{1} + \frac{1}{\cos\theta} - \cos\theta - \cancel{1} =$$

$$\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} =$$

$$\frac{1 - \cos^2\theta}{\cos\theta} =$$

$$\frac{\sin^2\theta}{\cos\theta} =$$

$$\frac{\sin\theta}{\cos\theta} \cdot \sin\theta = \tan\theta\sin\theta$$

d. $\left(1 + \frac{1}{\sin\theta}\right)(\sin\theta - \sin^2\theta) =$

$\sin\theta - \sin^2\theta + \frac{\sin\theta}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} =$

$\cancel{\sin}\theta - \sin^2\theta + 1 - \cancel{\sin}\theta =$

$1 - \sin^2\theta = \cos^2\theta$

Exercise 11D

#1 a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \boxed{\frac{24}{25}}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$= \frac{9}{25} - \frac{16}{25} = \boxed{\frac{-7}{25}}$

c. $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{24}{25}}{-\frac{7}{25}} = \frac{24}{\cancel{25}} \cdot \frac{\cancel{25}}{-7} = \boxed{-\frac{24}{7}}$

#2 a. $\cos A = \frac{1}{3}$

$\left(\frac{1}{3}\right)^2 + \sin^2 A = 1$

$\frac{1}{9} + \sin^2 A = \frac{9}{9}$

$\sin^2 A = \frac{8}{9}$

$\sin A = \pm\sqrt{\frac{8}{9}}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= \left(\frac{1}{3}\right)^2 - \left(\pm\sqrt{\frac{8}{9}}\right)^2$

$= \frac{1}{9} - \frac{8}{9} = \boxed{\frac{-7}{9}}$

b. $\sin\theta = \frac{-2}{3}$

$\cos^2\theta + \left(-\frac{2}{3}\right)^2 = 1$

$\cos^2\theta = \frac{5}{9}$

$\cos\theta = \pm\sqrt{\frac{5}{9}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\pm\sqrt{\frac{5}{9}}\right)^2 - \left(-\frac{2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9}}$

#3 $\sin\alpha = -\frac{2}{3}$ -AND- Q3

$\cos^2\alpha + \left(-\frac{2}{3}\right)^2 = 1$

$\cos^2\alpha = \frac{5}{9}$

$\cos\alpha = \pm\sqrt{\frac{5}{9}}$

a) Q3 : $\cos\alpha = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$

b) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$

$= \boxed{\frac{4\sqrt{5}}{9}}$

#8 a. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta =$

$\underbrace{\sin^2\theta + \cos^2\theta}_{1} + 2\sin\theta\cos\theta =$

$1 + 2\sin\theta\cos\theta =$

$1 + \sin 2\theta$

b. $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) =$

$(\cos^2\theta - \sin^2\theta)(1) =$

$\cos 2\theta$