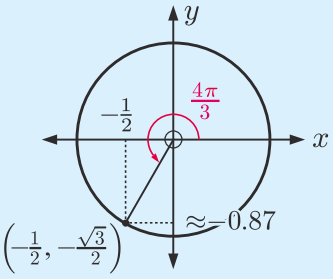
Example 14 Self Tutor

Find the exact values of $\sin\frac{4\pi}{3}$, $\cos\frac{4\pi}{3}$, and $\tan\frac{4\pi}{3}$.



$$\sin\left(\tfrac{4\pi}{3}\right) = -\tfrac{\sqrt{3}}{2}$$
$$\cos\left(\tfrac{4\pi}{3}\right) = -\tfrac{1}{2}$$
$$\tan\left(\tfrac{4\pi}{3}\right) = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

EXERCISE 8E

1 Use a unit circle diagram to find exact values for $\sin\theta$, $\cos\theta$, and $\tan\theta$ for θ equal to:

a $\frac{\pi}{4}$ **b** $\frac{3\pi}{4}$ **c** $\frac{7\pi}{4}$ **d** π **e** $\frac{-3\pi}{4}$

2 Use a unit circle diagram to find exact values for $\sin\beta$, $\cos\beta$, and $\tan\beta$ for β equal to:

a $\frac{\pi}{6}$ **b** $\frac{2\pi}{3}$ **c** $\frac{7\pi}{6}$ **d** $\frac{5\pi}{3}$ **e** $\frac{11\pi}{6}$

3 Find the exact values of:

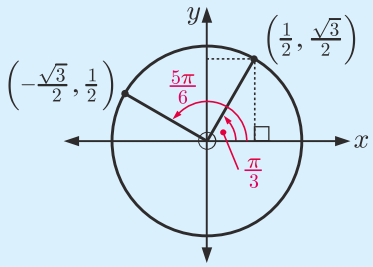
a $\cos 120°$, $\sin 120°$, and $\tan 120°$ **b** $\cos(-45°)$, $\sin(-45°)$, and $\tan(-45°)$

4 **a** Find the exact values of $\cos 90°$ and $\sin 90°$.

b What can you say about $\tan 90°$?

Example 15 Self Tutor

Without using a calculator, show that $8\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{5\pi}{6}\right) = -6$.



$$\sin\left(\tfrac{\pi}{3}\right) = \tfrac{\sqrt{3}}{2} \quad \text{and} \quad \cos\left(\tfrac{5\pi}{6}\right) = -\tfrac{\sqrt{3}}{2}$$
$$\therefore\ 8\sin\left(\tfrac{\pi}{3}\right)\cos\left(\tfrac{5\pi}{6}\right) = 8\left(\tfrac{\sqrt{3}}{2}\right)\left(-\tfrac{\sqrt{3}}{2}\right)$$
$$= 2(-3)$$
$$= -6$$

5 Without using a calculator, evaluate:

a $\sin^2 60°$ **b** $\sin 30° \cos 60°$ **c** $4\sin 60° \cos 30°$

d $1 - \cos^2\left(\frac{\pi}{6}\right)$ **e** $\sin^2\left(\frac{2\pi}{3}\right) - 1$ **f** $\cos^2\left(\frac{\pi}{4}\right) - \sin\left(\frac{7\pi}{6}\right)$

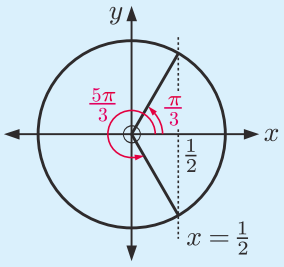
g $\sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{5\pi}{4}\right)$ **h** $1 - 2\sin^2\left(\frac{7\pi}{6}\right)$ **i** $\cos^2\left(\frac{5\pi}{6}\right) - \sin^2\left(\frac{5\pi}{6}\right)$

j $\tan^2\left(\frac{\pi}{3}\right) - 2\sin^2\left(\frac{\pi}{4}\right)$ **k** $2\tan\left(-\frac{5\pi}{4}\right) - \sin\left(\frac{3\pi}{2}\right)$ **l** $\dfrac{2\tan 150°}{1 - \tan^2 150°}$

Check all answers using your calculator.

Example 16 **Self Tutor**

Find all angles $0 \leqslant \theta \leqslant 2\pi$ with a cosine of $\frac{1}{2}$.



Since the cosine is $\frac{1}{2}$, we draw the vertical line $x = \frac{1}{2}$.

Because $\frac{1}{2}$ is involved, we know the required angles are multiples of $\frac{\pi}{6}$.

They are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

6 Find all angles between $0°$ and $360°$ with:

a a sine of $\frac{1}{2}$ **b** a sine of $\frac{\sqrt{3}}{2}$ **c** a cosine of $\frac{1}{\sqrt{2}}$

d a cosine of $-\frac{1}{2}$ **e** a cosine of $-\frac{1}{\sqrt{2}}$ **f** a sine of $-\frac{\sqrt{3}}{2}$

7 Find all angles between 0 and 2π (inclusive) which have:

a a tangent of 1 **b** a tangent of -1 **c** a tangent of $\sqrt{3}$

d a tangent of 0 **e** a tangent of $\frac{1}{\sqrt{3}}$ **f** a tangent of $-\sqrt{3}$

8 Find all angles between 0 and 4π with:

a a cosine of $\frac{\sqrt{3}}{2}$ **b** a sine of $-\frac{1}{2}$ **c** a sine of -1

9 Find θ if $0 \leqslant \theta \leqslant 2\pi$ and:

a $\cos\theta = \frac{1}{2}$ **b** $\sin\theta = \frac{\sqrt{3}}{2}$ **c** $\cos\theta = -1$ **d** $\sin\theta = 1$

e $\cos\theta = -\frac{1}{\sqrt{2}}$ **f** $\sin^2\theta = 1$ **g** $\cos^2\theta = 1$ **h** $\cos^2\theta = \frac{1}{2}$

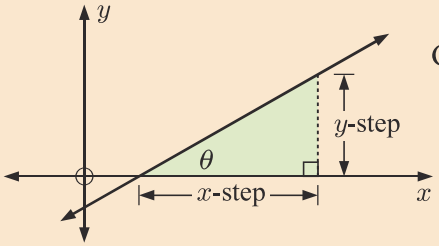
i $\tan\theta = -\frac{1}{\sqrt{3}}$ **j** $\tan^2\theta = 3$

10 Find *all* values of θ for which $\tan\theta$ is: **a** zero **b** undefined.

F THE EQUATION OF A STRAIGHT LINE

If a straight line makes an angle of θ with the positive x-axis then its gradient is $m = \tan\theta$.

Proof:



Gradient $m = \frac{y\text{-step}}{x\text{-step}}$

$= \tan\theta$