EXERCISE 8D.1

1 Find the possible exact values of $\cos\theta$ for:

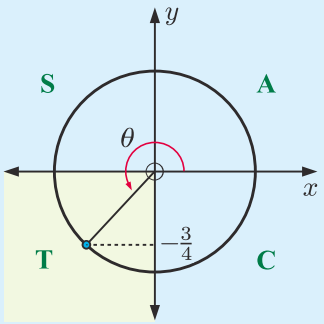
a $\sin\theta = \frac{1}{2}$ **b** $\sin\theta = -\frac{1}{3}$ **c** $\sin\theta = 0$ **d** $\sin\theta = -1$

2 Find the possible exact values of $\sin\theta$ for:

a $\cos\theta = \frac{4}{5}$ **b** $\cos\theta = -\frac{3}{4}$ **c** $\cos\theta = 1$ **d** $\cos\theta = 0$

Example 9 Self Tutor

If $\sin\theta = -\frac{3}{4}$ and $\pi < \theta < \frac{3\pi}{2}$, find $\cos\theta$ and $\tan\theta$. Give exact values.



Now $\cos^2\theta + \sin^2\theta = 1$

$\therefore \cos^2\theta + \frac{9}{16} = 1$

$\therefore \cos^2\theta = \frac{7}{16}$

$\therefore \cos\theta = \pm\frac{\sqrt{7}}{4}$

But $\pi < \theta < \frac{3\pi}{2}$, so θ is a quadrant 3 angle

$\therefore \cos\theta$ is negative.

$\therefore \cos\theta = -\frac{\sqrt{7}}{4}$

and $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$

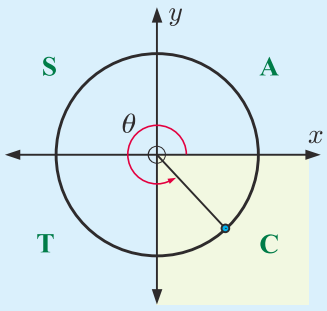
3 Without using a calculator, find:

a $\sin\theta$ if $\cos\theta = \frac{2}{3}$ and $0 < \theta < \frac{\pi}{2}$ **b** $\cos\theta$ if $\sin\theta = \frac{2}{5}$ and $\frac{\pi}{2} < \theta < \pi$

c $\cos\theta$ if $\sin\theta = -\frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$ **d** $\sin\theta$ if $\cos\theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$.

Example 10 Self Tutor

If $\tan\theta = -2$ and $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin\theta$ and $\cos\theta$. Give exact answers.



$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = -2$

$\therefore \sin\theta = -2\cos\theta$

Now $\sin^2\theta + \cos^2\theta = 1$

$\therefore (-2\cos\theta)^2 + \cos^2\theta = 1$

$\therefore 4\cos^2\theta + \cos^2\theta = 1$

$\therefore 5\cos^2\theta = 1$

$\therefore \cos\theta = \pm\frac{1}{\sqrt{5}}$

But $\frac{3\pi}{2} < \theta < 2\pi$, so θ is a quadrant 4 angle.

$\therefore$ $\cos\theta$ is positive and $\sin\theta$ is negative.

$\therefore \cos\theta = \frac{1}{\sqrt{5}}$ and $\sin\theta = -\frac{2}{\sqrt{5}}$.

4 **a** If $\sin x = \frac{1}{3}$ and $\frac{\pi}{2} < x < \pi$, find $\tan x$ exactly.

b If $\cos x = \frac{1}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\tan x$ exactly.

c If $\sin x = -\frac{1}{\sqrt{3}}$ and $\pi < x < \frac{3\pi}{2}$, find $\tan x$ exactly.

d If $\cos x = -\frac{3}{4}$ and $\frac{\pi}{2} < x < \pi$, find $\tan x$ exactly.

5 Find exact values for $\sin x$ and $\cos x$ given that:

a $\tan x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$ **b** $\tan x = -\frac{4}{3}$ and $\frac{\pi}{2} < x < \pi$

c $\tan x = \frac{\sqrt{5}}{3}$ and $\pi < x < \frac{3\pi}{2}$ **d** $\tan x = -\frac{12}{5}$ and $\frac{3\pi}{2} < x < 2\pi$

6 Suppose $\tan x = k$ where k is a constant and $\pi < x < \frac{3\pi}{2}$. Write expressions for $\sin x$ and $\cos x$ in terms of k.

FINDING ANGLES WITH PARTICULAR TRIGONOMETRIC RATIOS

From **Exercise 8C** you should have discovered that:

For θ in radians:

- $\sin(\pi - \theta) = \sin\theta$
- $\cos(\pi - \theta) = -\cos\theta$
- $\cos(2\pi - \theta) = \cos\theta$

We need results such as these, and also the periodicity of the trigonometric ratios, to find angles which have a particular sine, cosine, or tangent.

Example 11 **Self Tutor**

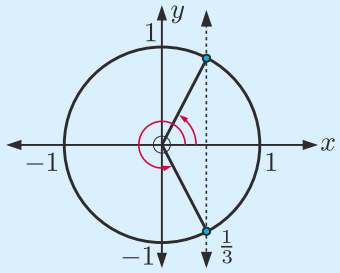
Find the two angles θ on the unit circle, with $0 \leqslant \theta \leqslant 2\pi$, such that:

a $\cos\theta = \frac{1}{3}$ **b** $\sin\theta = \frac{3}{4}$ **c** $\tan\theta = 2$

a Using technology,

$\cos\theta = \frac{1}{3}$

$\cos^{-1}(\frac{1}{3}) \approx 1.23$



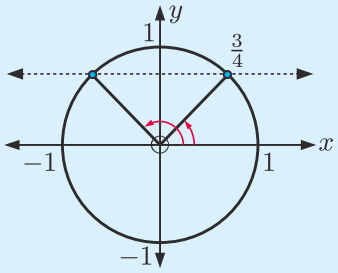
$\therefore\ \theta \approx 1.23$ or $2\pi - 1.23$

$\therefore\ \theta \approx 1.23$ or 5.05

b Using technology,

$\sin\theta = \frac{3}{4}$

$\sin^{-1}(\frac{3}{4}) \approx 0.848$



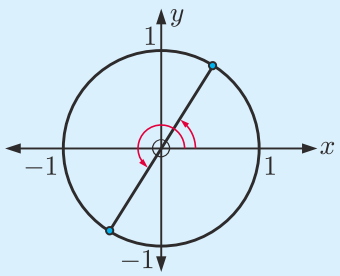
$\therefore\ \theta \approx 0.848$ or $\pi - 0.848$

$\therefore\ \theta \approx 0.848$ or 2.29

c Using technology,

$\tan\theta = 2$

$\tan^{-1}(2) \approx 1.11$



$\therefore\ \theta \approx 1.11$ or $\pi + 1.11$

$\therefore\ \theta \approx 1.11$ or 4.25