Example 9 Self Tutor

72% of union members are in favour of a certain change to their conditions of employment. A random sample of five members is taken. Find:

a the probability that three members are in favour of the change in conditions

b the probability that at least three members are in favour of the changed conditions

c the expected number of members in the sample that are in favour of the change.

Let X denote the number of members in the sample in favour of the changes.

$n = 5$, so $X = 0, 1, 2, 3, 4$, or 5, and $p = 72\% = 0.72$

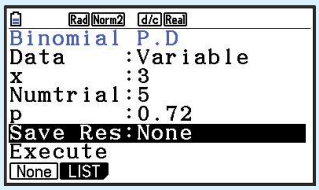
$\therefore \ X \sim \text{B}(5, 0.72)$.

a $\text{P}(X = 3)$

$= \binom{5}{3} (0.72)^3(0.28)^2$

≈ 0.293

Casio fx-CG20



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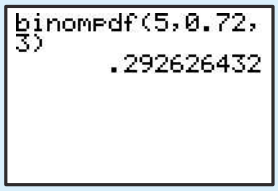

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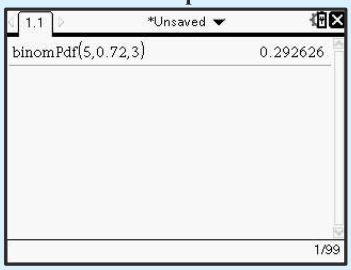

b $\text{P}(X \geqslant 3)$

≈ 0.862

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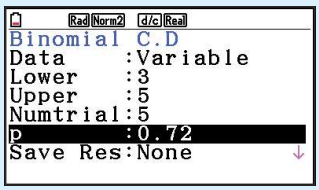

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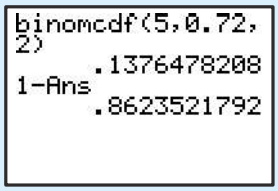

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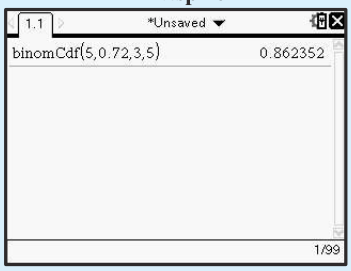

c $\text{E}(X) = np = 5 \times 0.72 = 3.6$ members

EXERCISE 23D.2

1 For which of these probability experiments does the binomial distribution apply? Justify your answers, using a full sentence.

a A coin is thrown 100 times. The variable is the number of heads.

b One hundred coins are each thrown once. The variable is the number of heads.

c A box contains 5 blue and 3 red marbles. I draw out 5 marbles, replacing the marble each time. The variable is the number of red marbles drawn.

d A box contains 5 blue and 3 red marbles. I draw out 5 marbles without replacement. The variable is the number of red marbles drawn.

e A large bin contains ten thousand bolts, 1% of which are faulty. I draw a sample of 10 bolts from the bin. The variable is the number of faulty bolts.

2 5% of electric light bulbs are defective at manufacture. If 6 bulbs are tested at random with each one being replaced before the next is chosen, determine the probability that:

a two are defective **b** at least one is defective.

3 In a multiple choice test there are 10 questions. Each question has 5 choices, one of which is correct. If 70% is the pass mark and Raj, who knows absolutely nothing about the subject, guesses each answer at random, determine the probability that he will pass.

4 At a manufacturing plant, 35% of the employees work night-shift. If 7 employees are each selected from the entire group at random, find the probability that:

a exactly 3 of them work night-shift **b** less than 4 of them work night-shift

c at least 4 of them work night-shift.

5 Records show that 6% of the items assembled on a production line are faulty.
A random sample of 12 items is selected with replacement. Find the probability that:

a none will be faulty **b** at most one will be faulty

c at least two will be faulty **d** less than four will be faulty.

6 There is a 5% chance that any apple in a crate will have a blemish. If a random sample of 25 apples is taken with replacement, find:

a the probability that exactly 2 of these have blemishes

b the probability that at least one has a blemish

c the expected number of apples that will have a blemish.

7 The local bus service does not have a good reputation. The 8 am bus will run late on average two days out of every five. For any week of the year taken at random, find the probability of the 8 am bus being on time:

a all 7 days **b** only on Monday **c** on any 6 days **d** on at least 4 days.

8 An infectious flu virus is spreading through a school. The probability of a randomly selected student having the flu next week is 0.3.

a Mr C has a class of 25 students.

i Calculate the probability that 2 or more students will have the flu next week.

ii If more than 20% of the students have the flu next week, a class test will have to be cancelled. What is the probability that the test will be cancelled?

b If the school has 350 students, find the expected number that will have the flu next week.

9 During a season, a basketball player has a 94% success rate in shooting from the free throw line. In one match the basketballer has 20 shots from the free throw line.

a Find the probability that the basketballer is successful on:

i all 20 throws **ii** at least 18 throws.

b Find the expected number of successful throws for the match.