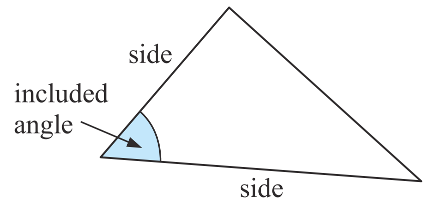
Summary:

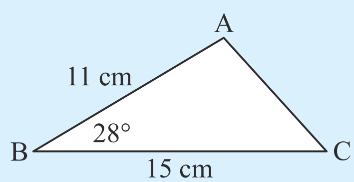
Given the lengths of two sides of a triangle and the angle between them (called the **included angle**), the area of the triangle is

a half of the product of two sides and the **sine of the included angle**.



Example 1

Find the area of triangle ABC:

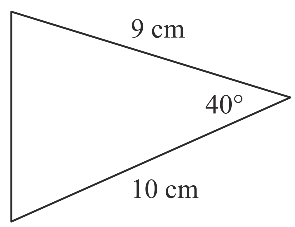


$$\text{Area} = \tfrac{1}{2}ac\sin B$$
$$= \tfrac{1}{2} \times 15 \times 11 \times \sin 28^o$$
$$\doteqdot 38.7 \text{ cm}^2$$

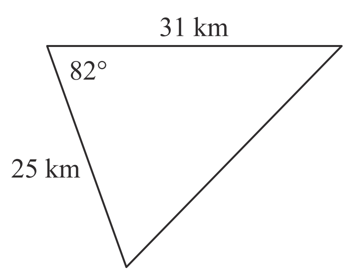
EXERCISE 12A

1 Find the area of:

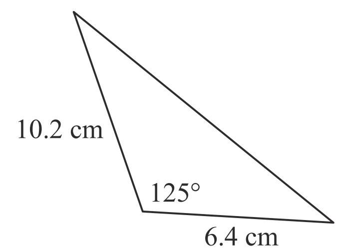
a



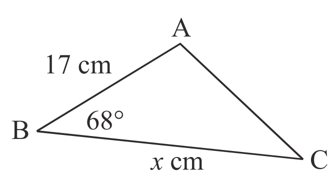
b



c



2 If triangle ABC has area 150 cm^2, find the value of x:



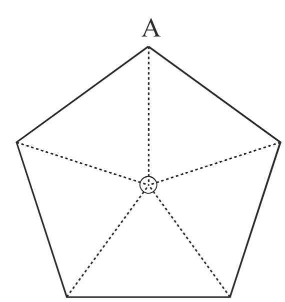
3 A parallelogram has two adjacent sides of length 4 cm and 6 cm respectively. If the included angle measures 52^o, find the area of the parallelogram.

4 A rhombus has side lengths 12 cm and an angle of 72^o. Find its area.

5 Find the area of a regular hexagon with sides of length 12 cm.

6 A rhombus has an area of 50 cm^2 and an internal angle of size 63^o. Find the length of its sides.

7 A regular pentagonal garden plot has centre of symmetry O and an area of 338 m^2. Find the distance OA.



D THE SINE RULE

The **sine rule** is a set of equations which connects the lengths of the sides of any triangle with the sines of the angles of the triangle. The triangle does not have to be right angled for the sine rule to be used.

> In any triangle ABC with sides a, b and c units in length, and opposite angles A, B and C respectively,
>
> $$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Proof: The area of any triangle ABC is given by $\frac{1}{2}bc\sin A=\frac{1}{2}ac\sin B=\frac{1}{2}ab\sin C$.

Dividing each expression by $\frac{1}{2}abc$ gives

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}.$$

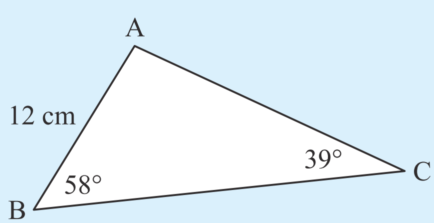
Note: The sine rule is used to solve problems involving triangles given either:

- **two angles** and **one side**, or
- **two sides** and a **non-included** angle.

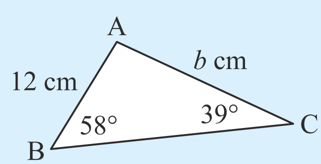
FINDING SIDES

Example 7

Find the length of AC correct to two decimal places.



By the sine rule



$\therefore \quad \dfrac{b}{\sin 58^o}=\dfrac{12}{\sin 39^o}$

$\therefore \quad b=\dfrac{12\times\sin 58^o}{\sin 39^o}$

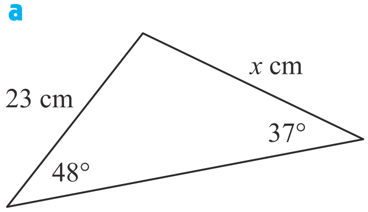
$\therefore \quad b \doteqdot 16.170\,74$

$\therefore$ AC is 16.2 cm long.

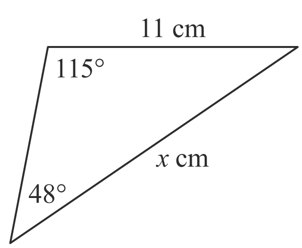
EXERCISE 12D.1

1 Find the value of x:

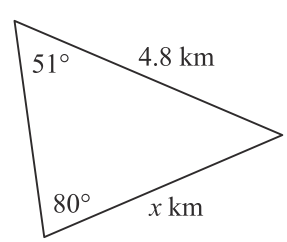
a



b



c



2 In triangle ABC find:

- **a** a if A = 63^o, B = 49^o and b = 18 cm
- **b** b if A = 82^o, C = 25^o and c = 34 cm
- **c** c if B = 21^o, C = 48^o and a = 6.4 cm

FINDING ANGLES

The problem of finding angles using the sine rule is more complicated because there may be two possible answers.

INVESTIGATION — THE AMBIGUOUS CASE

You will need a blank sheet of paper, a ruler, a protractor and a compass for the tasks that follow. In each task you will be required to construct triangles from given information.

Task 1: Draw AB = 10 cm. At A construct an angle of 30^o. Using B as centre, draw an arc of a circle of radius 6 cm. Let the arc intersect the ray from A at C. How many different positions may C have and therefore how many different triangles ABC may be constructed?

Task 2: As before, draw AB = 10 cm and construct a 30^o angle at A. This time draw an arc of radius 5 cm based on B. How many different triangles are possible?

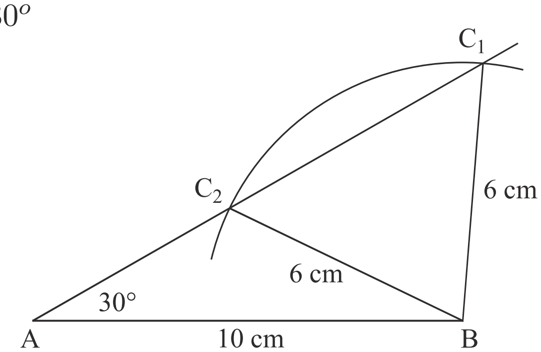
Task 3: Repeat, but this time draw an arc of radius 3 cm on B. How many different triangles are possible?

Task 4: Repeat with an arc of radius 12 cm from B. How many possible triangles?

In this investigation you should have discovered that when you are given two sides and a non-included angle there are a number of different possibilities. You could get two triangles, one triangle or it may be impossible to draw any triangles from the given data.

Let us consider the calculations involved in each of the cases of the investigation.

Task 1: Given: $c = 10$ cm, $a = 6$ cm, $A = 30^o$



Finding C: $$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\therefore \quad \sin C = \frac{c \sin A}{a}$$

$$\therefore \quad \sin C = \frac{10 \times \sin 30^o}{6} = 0.8333$$

Because $\sin\theta = \sin(180^o - \theta)$ there are two possible angles:

$$C = 56.44^o \quad or \quad 180^o - 56.44^o = 123.56^o$$

On your calculator check that the sin ratio of both of these angles is 0.8333.

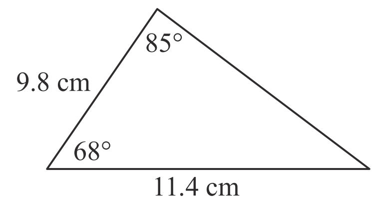
EXERCISE 12D.2

1 Triangle ABC has $\measuredangle B = 40^o$, $b = 8$ cm and $c = 11$ cm. Find the two possible values for angle C.

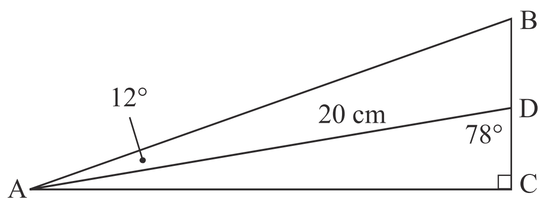
2 In triangle ABC, find the measure of:

a angle A if $a = 14.6$ cm, $b = 17.4$ cm and $\measuredangle \text{ABC} = 65^o$
b angle B if $b = 43.8$ cm, $c = 31.4$ cm and $\measuredangle \text{ACB} = 43^o$
c angle C if $a = 6.5$ km, $c = 4.8$ km and $\measuredangle \text{BAC} = 71^o$.

3 Is it possible to have a triangle with measurements as shown? Explain!

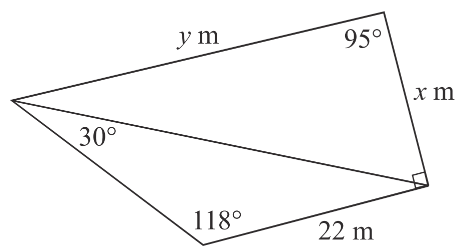


4



Find the magnitude of the angle ABC and hence BD in the given figure.

5 Find x and y in the given figure.



E USING THE SINE AND COSINE RULES

First decide which rule to use.

If the triangle is right angled then the trigonometric ratios or Pythagoras' Theorem can be used, and for some problems adding an extra line or two to the diagram may result in a right triangle.

However, if you have to choose between the sine and cosine rules, the following checklist may assist you.

Use the **cosine rule** when given	• three sides • two sides and an included angle.
Use the **sine rule** when given	• one side and two angles • two sides and a non-included angle (but beware of the *ambiguous case* which can occur when the smaller of the two given sides is opposite the given angle).