E | TRIGONOMETRIC EQUATIONS IN QUADRATIC FORM

Sometimes we may be given trigonometric equations in quadratic form.

For example, $2\sin^2 x + \sin x = 0$ and $2\cos^2 x + \cos x - 1 = 0$ are clearly quadratic equations where the variables are $\sin x$ and $\cos x$ respectively.

These equations can be factorised by quadratic factorisation and then solved for x.
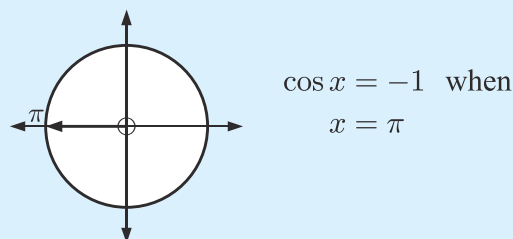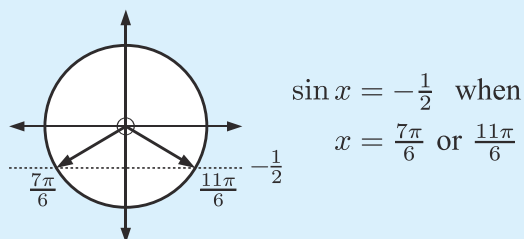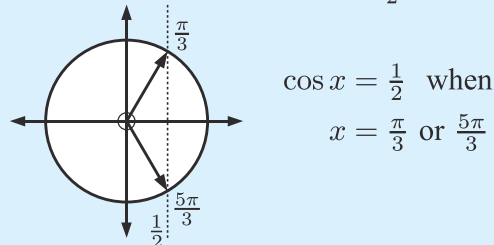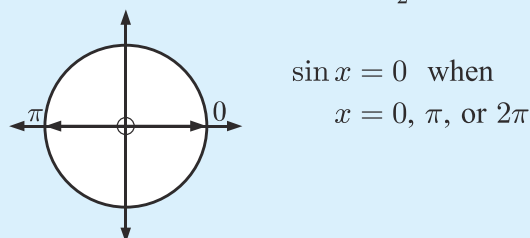

Example 19 ◀) **Self Tutor**

Solve for x on $0 \leqslant x \leqslant 2\pi$, giving your answers as exact values:

a $2\sin^2 x + \sin x = 0$
b $2\cos^2 x + \cos x - 1 = 0$

a
$$2\sin^2 x + \sin x = 0$$
$$\therefore \ \sin x(2\sin x + 1) = 0$$
$$\therefore \ \sin x = 0 \text{ or } -\tfrac{1}{2}$$

$\sin x = 0$ when
$x = 0, \pi, \text{ or } 2\pi$

$\sin x = -\tfrac{1}{2}$ when
$x = \tfrac{7\pi}{6} \text{ or } \tfrac{11\pi}{6}$

The solutions are:
$x = 0, \pi, \tfrac{7\pi}{6}, \tfrac{11\pi}{6}, \text{ or } 2\pi$.

b
$$2\cos^2 x + \cos x - 1 = 0$$
$$\therefore \ (2\cos x - 1)(\cos x + 1) = 0$$
$$\therefore \ \cos x = \tfrac{1}{2} \text{ or } -1$$

$\cos x = \tfrac{1}{2}$ when
$x = \tfrac{\pi}{3} \text{ or } \tfrac{5\pi}{3}$

$\cos x = -1$ when
$x = \pi$

The solutions are:
$x = \tfrac{\pi}{3}, \pi, \text{ or } \tfrac{5\pi}{3}$.

EXERCISE 11E

1 Solve for $0 \leqslant x \leqslant 2\pi$ giving your answers as exact values:

a $2\sin^2 x + \sin x = 0$
b $2\cos^2 x = \cos x$
c $2\cos^2 x + \cos x - 1 = 0$
d $2\sin^2 x + 3\sin x + 1 = 0$
e $\sin^2 x = 2 - \cos x$

2 Solve for $0 \leqslant x \leqslant 2\pi$ giving your answers as exact values:

a $\cos 2x - \cos x = 0$
b $\cos 2x + 3\cos x = 1$
c $\cos 2x + \sin x = 0$
d $\sin 4x = \sin 2x$
e $\sin x + \cos x = \sqrt{2}$
f $2\cos^2 x = 3\sin x$