Ex 11D #9

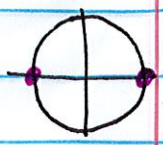
a. $\sin 2x + \sin x = 0$

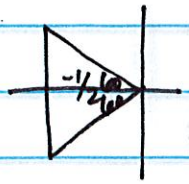
$2\sin x \cos x + \sin x = 0$

$\sin x (2\cos x + 1) = 0$

$\sin x = 0 \qquad 2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$





$x = 0, \pi, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

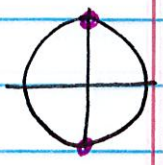
b. $\sin 2x - 2\cos x = 0$

$2\sin x \cos x - 2\cos x = 0$

$2\cos x (\sin x - 1) = 0$

$2\cos x = 0 \qquad \sin x - 1 = 0$

$\cos x = 0 \qquad \sin x = 1$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

c. $\sin 2x + 3\sin x = 0$

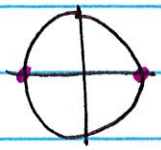
$2\sin x \cos x + 3\sin x = 0$

$\sin x (2\cos x + 3) = 0$

$\sin x = 0 \qquad 2\cos x + 3 = 0$

$\cos x = -\frac{3}{2}$

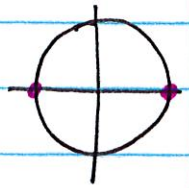
↑ not possible



$x = 0, \pi, 2\pi$

Ex 11 E #1, 2

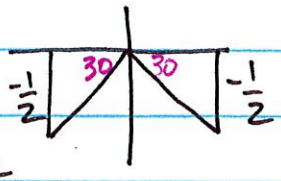
#1 a. $2\sin^2 x + \sin x = 0$



$\sin x(2\sin x + 1) = 0$

$\sin x = 0 \qquad 2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$



210°, 330°

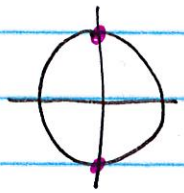
$\frac{7\pi}{6}, \frac{11\pi}{6}$

$x = 0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

b. $2\cos^2 x = \cos x$

* Always move everything to one side.

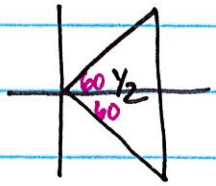
$2\cos^2 x - \cos x = 0$



$\cos x(2\cos x - 1) = 0$

$\cos x = 0 \qquad 2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$



60°, 300°

$\frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

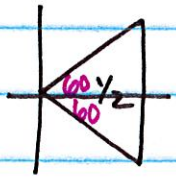
c. $2\cos^2 x + \cos x - 1 = 0$

Let $w = \cos x$

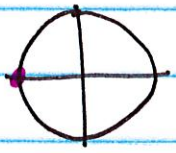
$2w^2 + w - 1 = 0$

$(2w - 1)(w + 1) = 0$

$(2\cos x - 1)(\cos x + 1) = 0$



$\cos x = \frac{1}{2} \qquad \cos x = -1$



$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

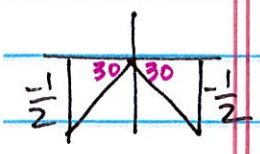
d. $2\sin^2 x + 3\sin x + 1 = 0$

Let $W = \sin x$

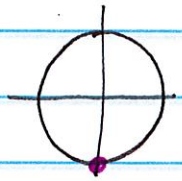
$2W^2 + 3W + 1 = 0$

$(2W+1)(W+1) = 0$

$(2\sin x + 1)(\sin x + 1) = 0$



$\sin x = \frac{-1}{2}$ $\sin x = -1$



$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

e. $\sin^2 x = 2 - \cos x$

$\sin^2 x + \cos x - 2 = 0$

$(1 - \cos^2 x) + \cos x - 2 = 0$

$-\cos^2 x + \cos x - 1 = 0$

$\cos^2 x - \cos x + 1 = 0$

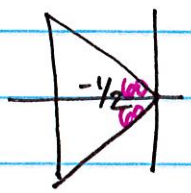
Let $W = \cos x$

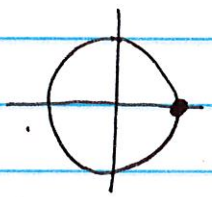
$W^2 - W + 1 = 0$

~~$(W-1)(W-1) = 0$~~ can't factor

$(-1)^2 - 4(1)(1) = -3$ ← discriminant is negative, so no real solutions.

#2 a. $\cos 2x - \cos x = 0$





$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

Let $W = \cos x$

$2W^2 - W - 1 = 0$

$(2W+1)(W-1) = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -1/2$ $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi$

b. $\cos 2x + 3\cos x = 1$

$\cos 2x + 3\cos x - 1 = 0$

$2\cos^2 x - 1 + 3\cos x - 1 = 0$

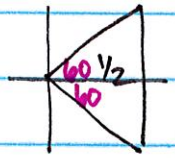
$2\cos^2 x + 3\cos x - 2 = 0$

Let $w = \cos x$

$2w^2 + 3w - 2 = 0$

$(2w - 1)(w + 2) = 0$

$(2\cos x - 1)(\cos x + 2) = 0$



$\cos x = \frac{1}{2}$ $\cos x = -2$ ↑ not possible

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

c. $\cos 2x + \sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

$-2\sin^2 x + \sin x + 1 = 0$

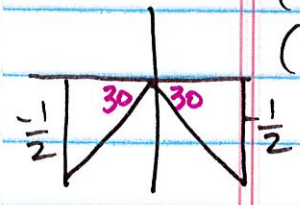
$2\sin^2 x - \sin x - 1 = 0$

Let $w = \sin x$

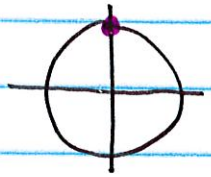
$2w^2 - w - 1 = 0$

$(2w + 1)(w - 1) = 0$

$(2\sin x + 1)(\sin x - 1) = 0$



$\sin x = -1/2$ $\sin x = 1$



$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

d. $\sin 4x = \sin 2x$

$\sin 2(2x) - \sin 2x = 0$

$2\sin 2x \cos 2x - \sin 2x = 0$

$\sin 2x (2\cos 2x - 1) = 0$

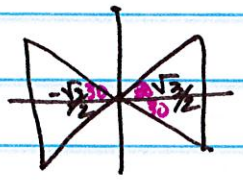
$\sin 2x = 0$ $\quad 2\cos 2x - 1 = 0$

$2\sin x \cos x = 0$ $\quad 2(2\cos^2 x - 1) - 1 = 0$

$\sin x \cos x = 0$ $\quad 4\cos^2 x - 2 - 1 = 0$

$\sin x = 0$ $\cos x = 0$ $\quad 4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$



$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

e. $\sin x + \cos x = \sqrt{2}$ $\quad \sqrt{2} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$\sin x + \cos x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$ $\quad \cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$

f. $2\cos^2 x = 3\sin x$

$2\cos^2 x - 3\sin x = 0$

$2(1 - \sin^2 x) - 3\sin x = 0$

$2 - 2\sin^2 x - 3\sin x = 0$

$2\sin^2 x + 3\sin x - 2 = 0$

Let $W = \sin x$

$2W^2 + 3W - 2 = 0$

$(2W - 1)(W + 2) = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = 1/2$ $\quad \sin x = -2$ ← not possible

$x = \frac{\pi}{6}, \frac{5\pi}{6}$