The **double angle formulae** are:

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

GRAPHING PACKAGE

Example 15 Self Tutor

Given that $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = -\frac{4}{5}$ find:

a $\sin 2\alpha$ **b** $\cos 2\alpha$

a $\sin 2\alpha$
$= 2\sin\alpha\cos\alpha$
$= 2(\frac{3}{5})(-\frac{4}{5})$
$= -\frac{24}{25}$

b $\cos 2\alpha$
$= \cos^2\alpha - \sin^2\alpha$
$= (-\frac{4}{5})^2 - (\frac{3}{5})^2$
$= \frac{7}{25}$

Example 16 Self Tutor

If $\sin\alpha = \frac{5}{13}$ where $\frac{\pi}{2} < \alpha < \pi$, find the exact value of $\sin 2\alpha$.

α is in quadrant 2, so $\cos\alpha$ is negative.

Now $\cos^2\alpha + \sin^2\alpha = 1$

$\therefore \cos^2\alpha + \frac{25}{169} = 1$

$\therefore \cos^2\alpha = \frac{144}{169}$

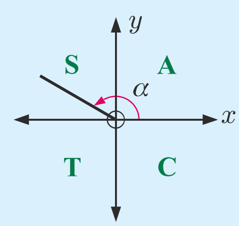
$\therefore \cos\alpha = \pm\frac{12}{13}$

$\therefore \cos\alpha = -\frac{12}{13}$

But $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\therefore \sin 2\alpha = 2(\frac{5}{13})(-\frac{12}{13})$

$= -\frac{120}{169}$



EXERCISE 11D

1 If $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$ find the exact values of:

a $\sin 2\theta$ **b** $\cos 2\theta$ **c** $\tan 2\theta$

2 **a** If $\cos A = \frac{1}{3}$, find $\cos 2A$. **b** If $\sin\phi = -\frac{2}{3}$, find $\cos 2\phi$.

3 If $\sin\alpha = -\frac{2}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$, find the exact value of:

a $\cos\alpha$ **b** $\sin 2\alpha$

4 If $\cos\beta = \frac{2}{5}$ where $270° < \beta < 360°$, find the exact value of:

a $\sin\beta$ **b** $\sin 2\beta$

Example 17 **Self Tutor**

If α is acute and $\cos 2\alpha = \frac{3}{4}$ find the exact values of: **a** $\cos\alpha$ **b** $\sin\alpha$.

a
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\therefore \quad \tfrac{3}{4} = 2\cos^2\alpha - 1$$
$$\therefore \quad \cos^2\alpha = \tfrac{7}{8}$$
$$\therefore \quad \cos\alpha = \pm\frac{\sqrt{7}}{2\sqrt{2}}$$
$$\therefore \quad \cos\alpha = \frac{\sqrt{7}}{2\sqrt{2}}$$
{as α is acute, $\cos\alpha > 0$}

b
$$\sin\alpha = \sqrt{1-\cos^2\alpha}$$
{as α is acute, $\sin\alpha > 0$}
$$\therefore \quad \sin\alpha = \sqrt{1-\tfrac{7}{8}}$$
$$\therefore \quad \sin\alpha = \sqrt{\tfrac{1}{8}}$$
$$\therefore \quad \sin\alpha = \frac{1}{2\sqrt{2}}$$

5 If α is acute and $\cos 2\alpha = -\frac{7}{9}$, find without a calculator: **a** $\cos\alpha$ **b** $\sin\alpha$.

Example 18 **Self Tutor**

Use an appropriate 'double angle formula' to simplify:

a $3\sin\theta\cos\theta$ **b** $4\cos^2 2B - 2$

a
$$3\sin\theta\cos\theta$$
$$= \tfrac{3}{2}(2\sin\theta\cos\theta)$$
$$= \tfrac{3}{2}\sin 2\theta$$

b
$$4\cos^2 2B - 2$$
$$= 2(2\cos^2 2B - 1)$$
$$= 2\cos 2(2B)$$
$$= 2\cos 4B$$

6 Find the exact value of $\left[\cos(\frac{\pi}{12}) + \sin(\frac{\pi}{12})\right]^2$.

7 Use an appropriate 'double angle' formula to simplify:

a $2\sin\alpha\cos\alpha$ **b** $4\cos\alpha\sin\alpha$ **c** $\sin\alpha\cos\alpha$
d $2\cos^2\beta - 1$ **e** $1 - 2\cos^2\phi$ **f** $1 - 2\sin^2 N$
g $2\sin^2 M - 1$ **h** $\cos^2\alpha - \sin^2\alpha$ **i** $\sin^2\alpha - \cos^2\alpha$
j $2\sin 2A\cos 2A$ **k** $2\cos 3\alpha\sin 3\alpha$ **l** $2\cos^2 4\theta - 1$
m $1 - 2\cos^2 3\beta$ **n** $1 - 2\sin^2 5\alpha$ **o** $2\sin^2 3D - 1$
p $\cos^2 2A - \sin^2 2A$ **q** $\cos^2(\frac{\alpha}{2}) - \sin^2(\frac{\alpha}{2})$ **r** $2\sin^2 3P - 2\cos^2 3P$

8 Show that:

a $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$ **b** $\cos^4\theta - \sin^4\theta = \cos 2\theta$

9 Solve exactly for x where $0 \leqslant x \leqslant 2\pi$:

a $\sin 2x + \sin x = 0$ **b** $\sin 2x - 2\cos x = 0$
c $\sin 2x + 3\sin x = 0$

GRAPHING PACKAGE

10 Use the double angle formula to show that:

a $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ **b** $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$