FACTORISING TRIGONOMETRIC EXPRESSIONS

Example 13 Self Tutor

Factorise: **a** $\cos^2\alpha - \sin^2\alpha$ **b** $\tan^2\theta - 3\tan\theta + 2$

a $\cos^2\alpha - \sin^2\alpha$
$= (\cos\alpha + \sin\alpha)(\cos\alpha - \sin\alpha)$ $\{a^2 - b^2 = (a+b)(a-b)\}$

b $\tan^2\theta - 3\tan\theta + 2$
$= (\tan\theta - 2)(\tan\theta - 1)$ $\{x^2 - 3x + 2 = (x-2)(x-1)\}$

Example 14 Self Tutor

Simplify:

a $\dfrac{2 - 2\cos^2\theta}{1 + \cos\theta}$ **b** $\dfrac{\cos\theta - \sin\theta}{\cos^2\theta - \sin^2\theta}$

a $\dfrac{2 - 2\cos^2\theta}{1 + \cos\theta}$
$= \dfrac{2(1 - \cos^2\theta)}{1 + \cos\theta}$
$= \dfrac{2(1+\cos\theta)(1 - \cos\theta)}{(1+\cos\theta)}$
$= 2(1 - \cos\theta)$

b $\dfrac{\cos\theta - \sin\theta}{\cos^2\theta - \sin^2\theta}$
$= \dfrac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$
$= \dfrac{1}{\cos\theta + \sin\theta}$

EXERCISE 11C.2

1 Factorise:

a $1 - \sin^2\theta$ **b** $\sin^2\alpha - \cos^2\alpha$ **c** $\tan^2\alpha - 1$
d $2\sin^2\beta - \sin\beta$ **e** $2\cos\phi + 3\cos^2\phi$ **f** $3\sin^2\theta - 6\sin\theta$
g $\tan^2\theta + 5\tan\theta + 6$ **h** $2\cos^2\theta + 7\cos\theta + 3$ **i** $6\cos^2\alpha - \cos\alpha - 1$

2 Simplify:

a $\dfrac{1 - \sin^2\alpha}{1 - \sin\alpha}$ **b** $\dfrac{\tan^2\beta - 1}{\tan\beta + 1}$ **c** $\dfrac{\cos^2\phi - \sin^2\phi}{\cos\phi + \sin\phi}$

d $\dfrac{\cos^2\phi - \sin^2\phi}{\cos\phi - \sin\phi}$ **e** $\dfrac{\sin\alpha + \cos\alpha}{\sin^2\alpha - \cos^2\alpha}$ **f** $\dfrac{3 - 3\sin^2\theta}{6\cos\theta}$

3 Show that:

a $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$ simplifies to 2
b $(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2$ simplifies to 13
c $(1 - \cos\theta)\left(1 + \dfrac{1}{\cos\theta}\right)$ simplifies to $\tan\theta\sin\theta$
d $\left(1 + \dfrac{1}{\sin\theta}\right)(\sin\theta - \sin^2\theta)$ simplifies to $\cos^2\theta$