## EXERCISE 10C

**1** Below is a table which shows the mean monthly maximum temperatures for a city in Greece.

| *Month* | Jan | Feb | Mar | Apr | May | Jun | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Temperature* (°C) | 15 | 14 | 15 | 18 | 21 | 25 | 27 | 26 | 24 | 20 | 18 | 16 |

**a** Use a sine function of the form $T \approx a\sin(b(t-c))+d$ to model the data. Find good estimates of the constants $a$, $b$, $c$ and $d$ without using technology. Use Jan $\equiv 1$, Feb $\equiv 2$, and so on.

**b** Use technology to check your answer to **a**. How well does your model fit?

**2** The data in the table shows the mean monthly temperatures for Christchurch, New Zealand.

| *Month* | Jan | Feb | Mar | Apr | May | Jun | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Temperature* (°C) | 15 | 16 | $14\frac{1}{2}$ | 12 | 10 | $7\frac{1}{2}$ | 7 | $7\frac{1}{2}$ | $8\frac{1}{2}$ | $10\frac{1}{2}$ | $12\frac{1}{2}$ | 14 |

**a** Find a sine model for this data in the form $T \approx a\sin(b(t-c))+d$. Assume Jan $\equiv 1$, Feb $\equiv 2$, and so on. Do not use technology.

**b** Use technology to check your answer to **a**.

**3** At the Mawson base in Antarctica, the mean monthly temperatures for the last 30 years are:

| *Month* | Jan | Feb | Mar | Apr | May | Jun | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Temperature* (°C) | 0 | 0 | $-4$ | $-9$ | $-14$ | $-17$ | $-18$ | $-19$ | $-17$ | $-13$ | $-6$ | $-2$ |

**a** Find a sine model for this data without using technology. Use Jan $\equiv 1$, Feb $\equiv 2$, and so on.

**b** How appropriate is the model?

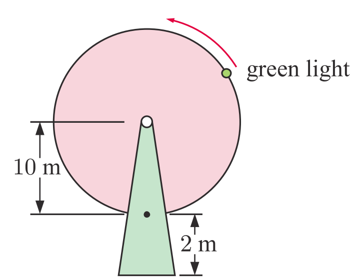
**4** Some of the largest tides in the world are observed in Canada's Bay of Fundy. The difference between high and low tides is 14 metres, and the average time difference between high tides is about 12.4 hours.

**a** Find a sine model for the height of the tide $H$ in terms of the time $t$.

**b** Sketch the graph of the model over one period.

**5** Revisit the **Opening Problem** on page **232**.

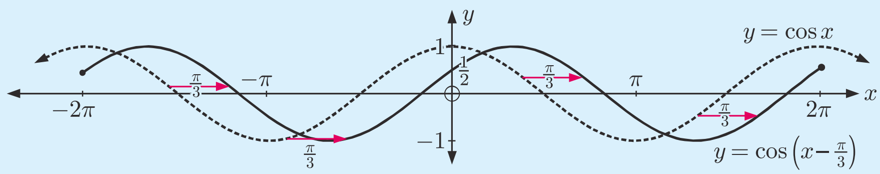
The wheel takes 100 seconds to complete one revolution. Find the sine model which gives the height of the light above the ground at any point in time. Assume that at time $t = 0$, the light is at its lowest point.



**Example 4** — Self Tutor

On the same set of axes graph $y = \cos x$ and $y = \cos\left(x - \frac{\pi}{3}\right)$ for $-2\pi \leqslant x \leqslant 2\pi$.

$y = \cos\left(x - \frac{\pi}{3}\right)$ is a horizontal translation of $y = \cos x$ through $\frac{\pi}{3}$ units to the right.



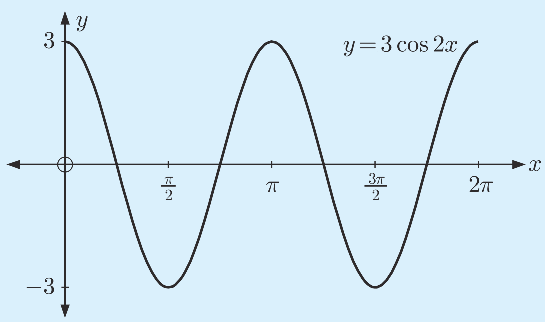

**Example 5** — Self Tutor

Without using technology, sketch the graph of $y = 3\cos 2x$ for $0 \leqslant x \leqslant 2\pi$.

Notice that $a = 3$, so the amplitude is $|3| = 3$.

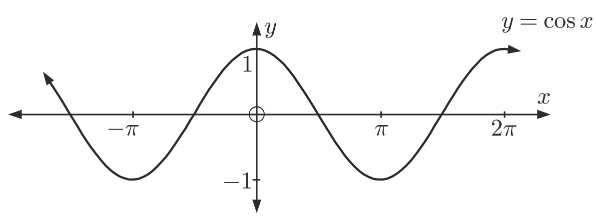
$b = 2$, so the period is $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$.

To obtain this from $y = \cos x$, we have a vertical stretch with scale factor 3 followed by a horizontal stretch with scale factor $\frac{1}{2}$, as the period has been halved.




## EXERCISE 10D

**1** Given the graph of $y = \cos x$, sketch the graphs of:




- **a** $y = \cos x + 2$
- **b** $y = \cos x - 1$
- **c** $y = \cos\left(x - \frac{\pi}{4}\right)$
- **d** $y = \cos\left(x + \frac{\pi}{6}\right)$
- **e** $y = \frac{2}{3}\cos x$
- **f** $y = \frac{3}{2}\cos x$
- **g** $y = -\cos x$
- **h** $y = \cos\left(x - \frac{\pi}{6}\right) + 1$
- **i** $y = \cos\left(x + \frac{\pi}{4}\right) - 1$
- **j** $y = \cos 2x$
- **k** $y = \cos\left(\frac{x}{2}\right)$
- **l** $y = 3\cos 2x$

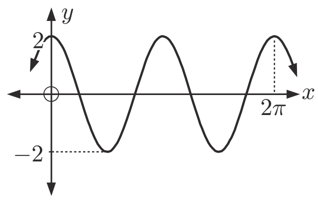
**2** Without graphing them, state the periods of:

- **a** $y = \cos 3x$
- **b** $y = \cos\left(\frac{x}{3}\right)$
- **c** $y = \cos\left(\frac{\pi}{50}x\right)$

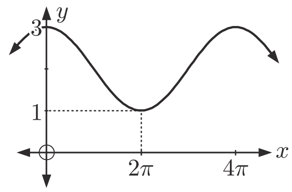
**3** The general cosine function is $y = a\cos(b(x - c)) + d$.
State the geometrical significance of $a$, $b$, $c$, and $d$.

**4** Find the cosine function shown in the graph:

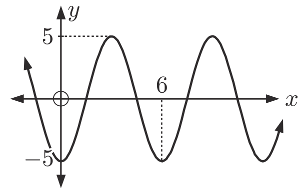
**a**




**b**




**c**

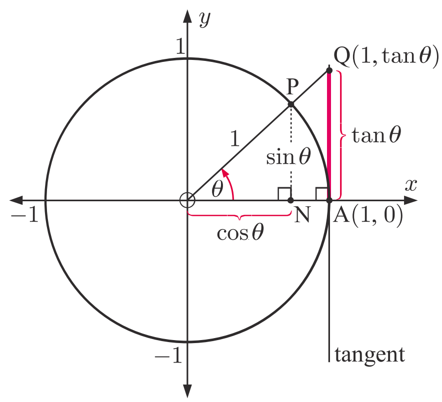


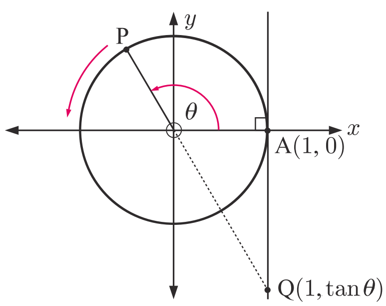

# E THE TANGENT FUNCTION

We have seen that if $P(\cos\theta, \sin\theta)$ is a point which is free to move around the unit circle, and if [OP] is extended to meet the tangent at $A(1, 0)$, the intersection between these lines occurs at $Q(1, \tan\theta)$.

This enables us to define the **tangent function**

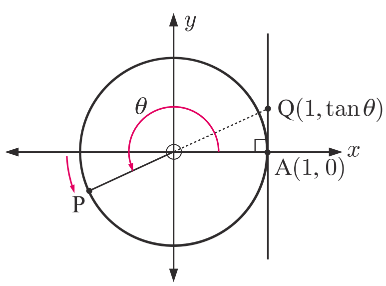
$$\tan\theta = \frac{\sin\theta}{\cos\theta}.$$



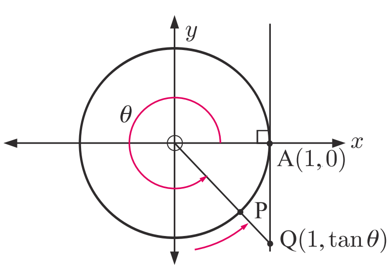




For $\theta$ in quadrant 2, $\sin\theta$ is positive and $\cos\theta$ is negative and so $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ is negative.

As before, [OP] is extended to meet the tangent at A at $Q(1, \tan\theta)$.




For $\theta$ in quadrant 3, $\sin\theta$ and $\cos\theta$ are both negative and so $\tan\theta$ is positive. This is clearly demonstrated as Q is back above the $x$-axis.




For $\theta$ in quadrant 4, $\sin\theta$ is negative and $\cos\theta$ is positive. $\tan\theta$ is again negative.